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Cost Risk Allocation

Objectives, Tendencies and Limitations

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- Los Angeles ■ Washington, D.C. ■ Boston ■ Chantilly ■ Huntsville ■ Dayton ■ Santa Barbara
- Albuquerque ■ Colorado Springs ■ Columbus ■ Ft. Meade ■ Ft. Monmouth ■ Montgomery ■ Ogden ■ Patuxent River ■ Pensacola ■ San Diego
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- **Overview**
2-3 minutes
- **What Is Cost Risk Allocation?**
10-15 minutes
- **Defining The Threat**
10-15 minutes
- **Minimizing Average Budget Overrun**
15-20 minutes
- **Minimizing Budget Overrun Semi-Variance**
20-30 minutes
- **In Conclusion**
2-3 minutes

Proverb

Knowledge is better
than blind practice.

-Fortune Cookie

Lucky numbers: 7 9 23 36 41, 19





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What Is Cost Risk Allocation?





A Risk By Any Other Name



Cost

The price paid to acquire, produce, accomplish, or maintain anything

Dictionary.com



Risk

The possibility of suffering harm or loss; danger

American Heritage Dictionary

Cost Risk Allocation

A process by which costs of subordinate WBS elements are allotted such that they sum to the parent cost at the selected cost risk

(working definition)

Cost Risk Consequence

The average additional cost suffered

(working definition)

Cost Risk

The probability of incurring additional cost to the budget

Dictionary.com

Allocate

To distribute according to a plan; allot

American Heritage Dictionary

Risk Dollars

The amount of funds needed to bring the TBE value up to a selected probability level

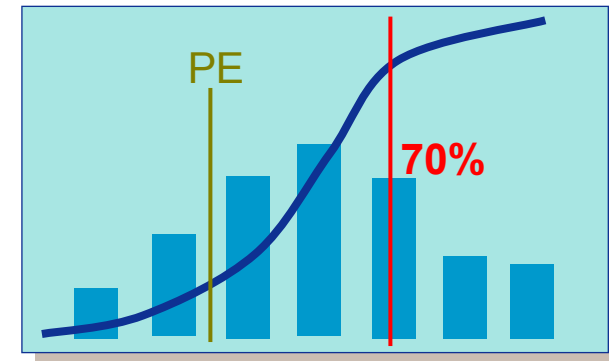
AFCAA CRH

Allocate Risk Dollars

To distribute risk dollars back to WBS elements
(paraphrase of presentation title of S. Book)

Uncertainty Is Understanding

- **Point Estimate Has No Context on Its Own**
 - How **precise** is our model?
 - How likely will we beat the P.E.?
 - What elements drive the **uncertainty**?
- **Cost Uncertainty Analysis...**
 - Quantifies **precision** of the model
 - Identifies ranges of **likely costs**
 - Reveals worrisome elements
- **However, Uncertainty *Doesn't* Add Up**
 - Accountants don't like this fact
 - Managers want an answer that they understand
 - Hard to compare against execution progress
- **Allocated Costs Add Up (just like P.E. and Mean)**
 - More **statistically meaningful** than point estimate
- **But Beware**
 - **Cost risk** of elements will **change** if their cost changes
 - Allocated estimate loses context of model **precision**
- **But What is a Good Cost Risk Allocation Method?**
 - It ultimately comes down to priorities



Cost = \$10,235,329.88



Cost = \$10M - \$12M



\$5.2M + \$6.3M = \$11.5M
 @73% @73% @80%

■ **First, Define What is Important:** (*may conflict*)

- Minimizing overruns that may occur
- Reducing chance of a budget overrun
- Protecting important systems from failure
- Meeting schedule demands
- Identifying money flow problems
- Tracking well to EVM during execution
- Etc.

■ **Next, Figure Out What You Can Manage:**

- Identifying and mitigating risk
- Holding funds in reserve
- Schedule and scope
- Etc.

■ **And What You Can't Manage:**

- Due to legal issues (color of money)
- Due to bureaucracy (approval and reporting)
- Due to project inertia (contracts and penalties)
- Etc.

Proverb

Digging a hole in the right
place is more important than
digging the hole right.



■ Our Ultimate Goal Is Project Success

- A good start means better chance of success
- Helps our manager make informed decisions

■ Our Realistic Goal Is Getting WBS to Add

- For whatever reason...
 - ... we must capture risk dollars in line items
 - ... we cannot show a reserve line

■ A Cost Risk Allocation Scheme...

- ...should reliably optimize what **concerns** us

■ Cost Risk Allocation Is a **Limited** Tool

- **Fails to capture** important issues that impact budget viability...
 - ... schedule risk, money flow, contract vehicle, risk mitigation, etc.



Proverb

When all you own is a hammer
everything looks like a nail.

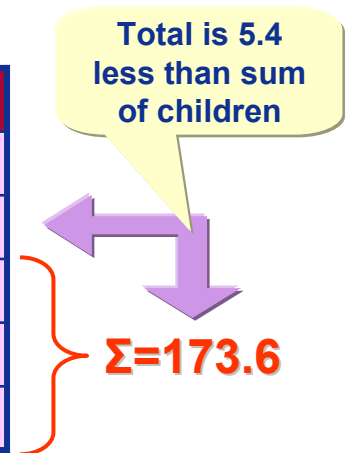
Could
our
model
capture
these?

The First Rule of Allocation

Perform cost risk allocation only when the WBS
must sum to a budget at a specified cost risk.

- **Ex: Allocate Air Vehicle for 25% Cost Risk**
 - i.e., 75% probability of being under budget
- **What is the “correct” way?**
 - **Semantically** correct as long as WBS adds up
- **Compare four methods**
 - (bad) Subtract 5.4 from largest elements
 - (bad) Subtract 1.8 from each element
 - (good) Minimize average size of cost overrun
 - (good) Minimize semi-variance (*explained later*)

Uncertainty Statistics	
WBS/CES	75.0%
Air Vehicle	168.2
Design & Dev.	34.7
Prototypes	18.6
Software	120.3



		Four Cost Risk Allocation Methods			
WBS/CES	Point Estimate	Subtract From Largest	Subtract 1.8 From Each	Average Overrun	Overrun Variance
Air Vehicle	111.5 (32%)	168.2 (75%)	168.2 (75%)	168.2 (75%)	168.2 (75%)
Design & Dev.	25.0 (25%)	34.7 (75%)	32.9 (67%)	34.0 (72%)	29.9 (54%)
Prototypes	9.7 (20%)	18.6 (75%)	16.8 (66%)	18.1 (72%)	14.6 (54%)
Software	76.8 (41%)	114.9 (71%)	118.5 (74%)	116.1 (72%)	123.7 (77%)



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Defining The Threat



An overrun may be a symptom of project illness

threat \propto *overrun*

threat \propto *overrun*²

The “Cost Camp”

■ Do You Believe?

- Your level of **angst** increases as overrun increases
- Subsystems should meet their budget **regardless of cost**
- The **percentage** of overrun defines the **threat** of failure
- Allocation should be **proportional** to the cost **risk**

The “Variance Camp”

■ Do You Believe?

- Your level of **angst** rapidly **accelerates** as overrun increases
- Less costly subsystems are **less important** to stay within budget
- The **dollar amount** of the overrun determines the **threat** of failure
- Allocation should be in proportion to the **square** of the cost **risk**

The risk of project failure encompasses more than a cost overrun

Proverb

Expenses grow to fill the budget.

Trip To The Mall

*You give Ben and Alice each \$15 for a CD.
How much change do you get back?*



*Ben paid \$12. Alice needs \$2 more.
Did you overrun by \$2 or recover \$1?*



*A cost model reports that you get \$1 back.
In our world, you need \$2 more to succeed.*

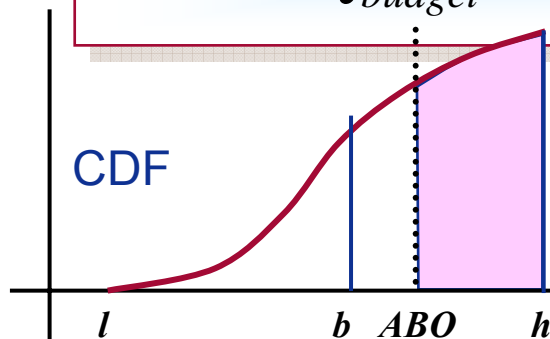
Proposed Definitions

Average Budget Overrun (ABO)

The cost risk consequence of a budget assigned to an element

$$ABO = \int_{budget}^{\infty} (c - budget) f(c) dc$$

Where,
 c is a potential cost
 $f(c)$ is the element's PDF

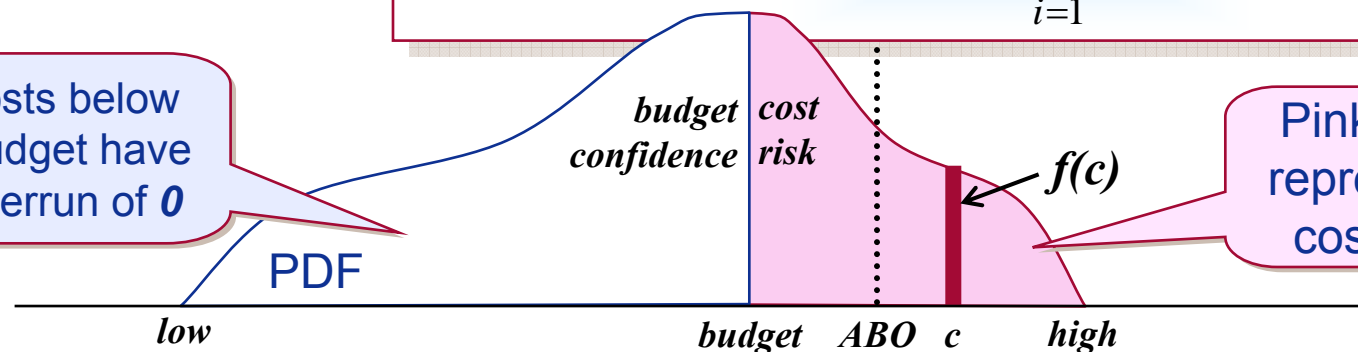


Total Average Budget Overrun (TABO)

The cost risk consequence for a sum of elements given money from under-budget elements cannot be recovered

$$TABO = \sum_{i=1}^n ABO_i$$

All costs below the budget have an overrun of 0



Pink area represents cost risk

Proposed Definitions

Budget Overrun Semi-Variance (BOSV)

A measure of risk consequence using the squares of each potential cost risk consequence weighted by probability of occurrence

$$BOSV = v^2 = \int_{budget}^{\infty} (c - budget)^2 f(c) du$$

Where,
 c is a potential total cost
 $f(c)$ is the element's PDF

Total Budget Overrun Semi-Variance (TBOSV)

A measure of risk consequence for a sum of elements including the impact of pairwise correlations among elements

$$TBOSV = \sum_{i=1}^n \sum_{j=1}^n \rho_{i,j} \sqrt{BOSV_i BOSV_j}$$

Where,
 ρ_{ij} is the correlation between i and j
 ρ represents a full correlation matrix
BOSV is Budget Overrun Semi-Variance

Look familiar?
Analogous to
variance.

$$\sigma^2 = \sum_{i=1}^n \sigma_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n \rho_{i,j} \sigma_i \sigma_j$$

Where,
 ρ_{ij} is the correlation between i and j
 σ^2 represents the variance for i



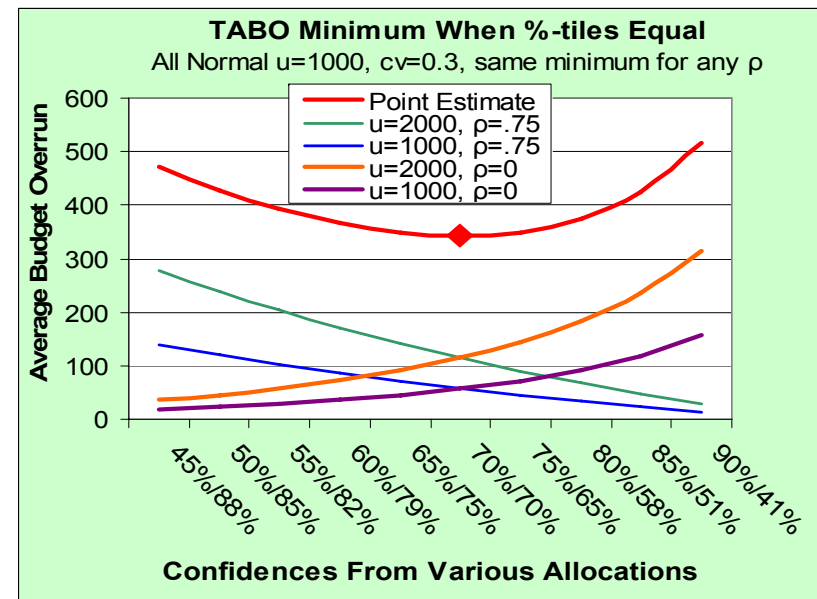
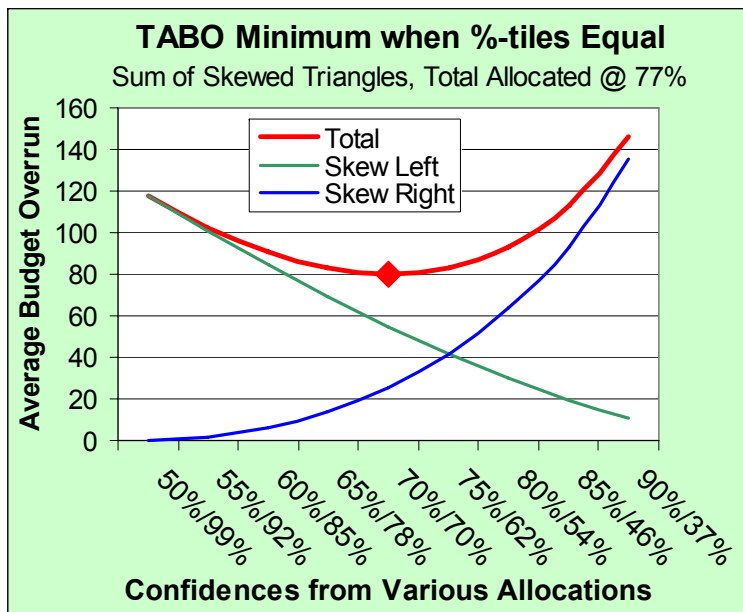
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Minimizing Average Cost Overrun





- **Charts show simulation results for sum of skewed and sum of correlated elements**
 - The total budget was 2370 (*77%-tile*) & 6945 (*81%-tile*) for respective charts, TABO in red
 - The uncertainty levels of A, B were altered by *increments of 5%* and ABO plotted
 - The uncertainty levels for C, D also displayed on X-axis for reference and ABO plotted
 - The total average budget overrun was plotted for each pair of element confidence levels
- **Result: Total average budget overrun was minimum when element confidences were equal**



WBS	Distribution	Low	Mode	High	Allocated	Alloc %-Tile
Total					2370	77%
Skew Left	Triangular	1000	2000	2000	1450	70%
Skew Right	Triangular	1000	1000	2000	920	70%

WBS	Distribution	Mean	CV	Allocated	Alloc %-Tile
Total		4000		6945	81%
A (Cor w/ B)	Normal	1000	0.2	2315	70%
B (Cor w/ A)	Normal	1000	0.2	1158	70%
C (Ind.)	Normal	1000	0.2	2315	70%
D (Ind.)	Normal	1000	0.2	1158	70%

Optimizing the “Cost Camp” Way...

- **When Allocating to Minimize the Total’s Average Budget Overrun...**
 - ...everything is already captured in the uncertainty statistics...
 - ...so don’t worry about integrating additional measures into method

Minimal Total Average Budget Overrun

Allocate so that all elements receiving funds end up at the same confidence level.

Negative
Correlation?

- **The Only Decisions to Make Are...**

- Where to **allocate from** – this should be where you can **manage funds**
- Where to **allocate to** – usually the **lowest level** WBS you are reporting
 - Also reasonable to allocate to **immediate children** and work up the WBS
- How **precise** to be with uncertainty levels (*why is explained later*)
 - About **±1% is fine** – after that round and report

i.e. move money
around WBS

Allocation Destination

■ Determine...

- ... WBS detail to report
- ... Where to allocate from
 - i.e., where you manage funds
- ... Where to allocate to
 - i.e., who is adjusted

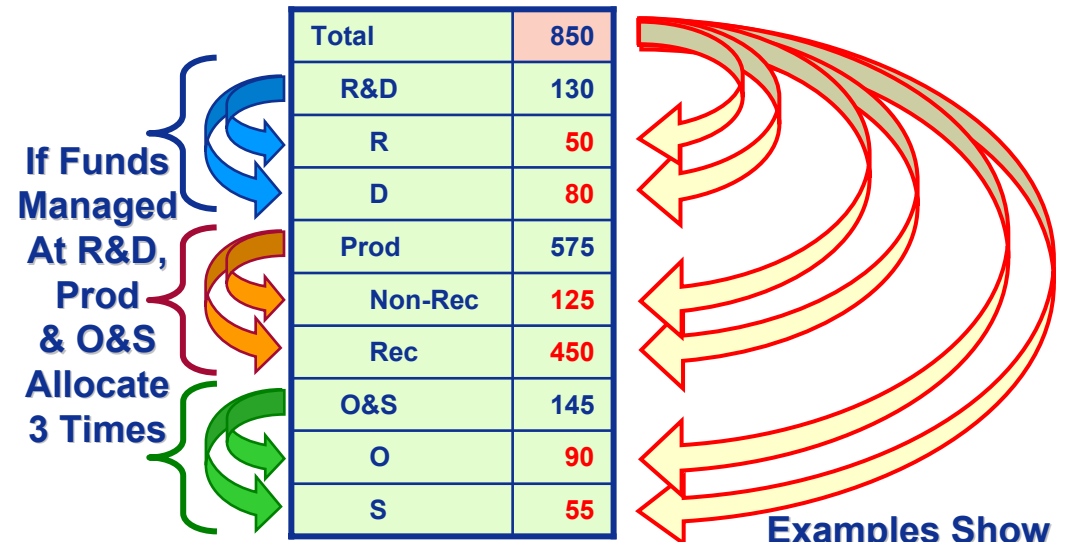
Multi-Tier Allocation Options:

■ 1) Allocate to lowest WBS

- And then sum up WBS
- ↑ One step process is easier to implement
- ↓ Mid-WBS values change if level of detail changes

■ 2) Allocate down WBS

- Allocate from total to immediate children
- And then, allocate from child to its grandchildren, etc.
- ↑ Keeps values consistent if report detail changes
- ↓ More steps to perform



Examples Show Funds Managed At Total

Total	850
R&D	126
R	46
D	80
Prod	524
Non-Rec	128
Rec	456
O&S	145
O	90
S	148



Adjusting Once for Tot. Ave. Budget Overrun:

(Easy to do and results close to optimal)

$$delta = budget_{total} - \sum_{i=1}^n cost_i$$

$$budget_i = cost_i + delta \frac{\sigma_i}{\sum_{j=1}^n \sigma_j}$$

Where,

$budget_{total}$ is the target cost for total for desired cost risk

$cost_i$ is row i 's cost with the same cost risk as **$budget_{total}$**

$delta$ is the amount to distribute among rows

$budget_i$ is the new, adjusted cost for row i after allocation

σ_i is the standard deviation of row i

Replace σ for the *square root of BOSV* if you feel like calculating it

Recursive Formula:

(For penny pinchers)

$$pct_0 = F_T(budget_{total})$$

$$cost_{r,i} = F_r^{-1}(pct_i)$$

$$delta_i = budget_{total} - \sum_{r=1}^n cost_{r,i}$$

$$budget_{r,i+1} = cost_{r,i} + \frac{delta_i \sigma_r}{\sum_{j=1}^n \sigma_j}$$

$$pct_{i+1} = \frac{1}{n} \sum_{r=1}^n F_r(budget_{r,i+1})$$

Where,

$budget_{total}$ is the desired total cost

pct_i is percentile of the rows to sum

$delta_i$ is the amount to distribute

$cost_{r,i}$ is the cost for row r at **$conf_i$**

σ_r is the standard deviation of row r

$F_r(v)$ is the CDF for row r

$F_T(v)$ is the CDF for the total

$F_r^{-1}(c)$ is the inverse CDF for row r

Calculation Example When Allocating to Lowest Reported WBS Level:


- Step 1: Pick cost risk of 25% (75%-tile)= \$608.94M (*this assumes we manage funds at total*)
- Step 2: Choose where to allocate to... 3rd level WBS elements (*lowest reported level*)
- Step 3: Calculate *delta*: Sum at @ 75% = 625.98 - *budget* = -17.04
- Step 4: Prorate *delta* for each element weighted by standard deviation (*or TABO*)
- Step 5: Determine confidence levels for each element's cost
- Step 6: If percentiles aren't close enough, use the weighted mean of the new levels as your next percentile, pct_{i+1} , and then return to step 3. (*Twice through is sufficient*)

WBS/CES	75% -Tile	Std Dev	Calculate Adjustment	Allocated	New %-Tiles
Total (\$M)	\$608.94			\$608.94	75.0%
Procurement	\$385.66			\$393.92	75.2%
Manufacturing (Air Force)	\$272.67	\$ 68.80	$-17 * 69 / 188 = -6.24$	\$266.43	72.2%
Ground Station LRIP Support	\$0.88	\$ 0.25	$-17 * 0.25 / 188 = -0.02$	\$0.86	72.7%
Transportation (AF)	\$2.00	\$ 0.57	$-17 * 0.57 / 188 = -0.05$	\$1.95	72.7%
Manufacturing (Army)	\$125.87	\$ 29.59	$-17 * 29 / 188 = -2.68$	\$123.19	72.2%
Transportable Ground Stations	\$0.91	\$ 0.24	$-17 * 0.24 / 188 = -0.02$	\$0.89	72.5%
Transportation (Army)	\$0.60	\$0.00	$-17 * 0 / 188 = -0.00$	\$0.60	
Quality Control	\$10.78	\$ 4.27	$-17 * 4.3 / 188 = -0.39$	\$10.39	72.7%
SEPM	\$212.27	\$ 84.14	$-17 * 84 / 188 = -7.64$	\$204.63	72.5%
SUM OF CHILDREN (\$M)	\$625.98	\$187.86	-\$17.04 to distribute	\$608.94	75.0%

Close
Enough

■ Calculation Example when Stepping Down WBS:

- Step 1: Pick project budget = \$608.9M (75% percentile)
- Step 2: Allocate budget for 1st Level to 2nd level WBS items (*its immediate children*)
 - Step 2-1: Calculate Δ_1 : *budget* of 608.9 - Sum at @ 75% of 616.6 = -7.7
 - Step 2-2: Prorate Δ_1 for each element weighted by standard deviation (or TABO)
 - Step 2-3: If percentiles aren't close enough, use weight mean of percentiles and repeat step 2
- Step 3: Take allocated budget for each 2nd level WBS element and allocate to 3rd level
 - Repeat steps 2-1 through 2-3 for each 2nd level *budget*₂, using *budget*₂ - sum of children @73.7%
- Step 4: If report contains 4th+ level WBS, Repeat step 3 for elements @ each level

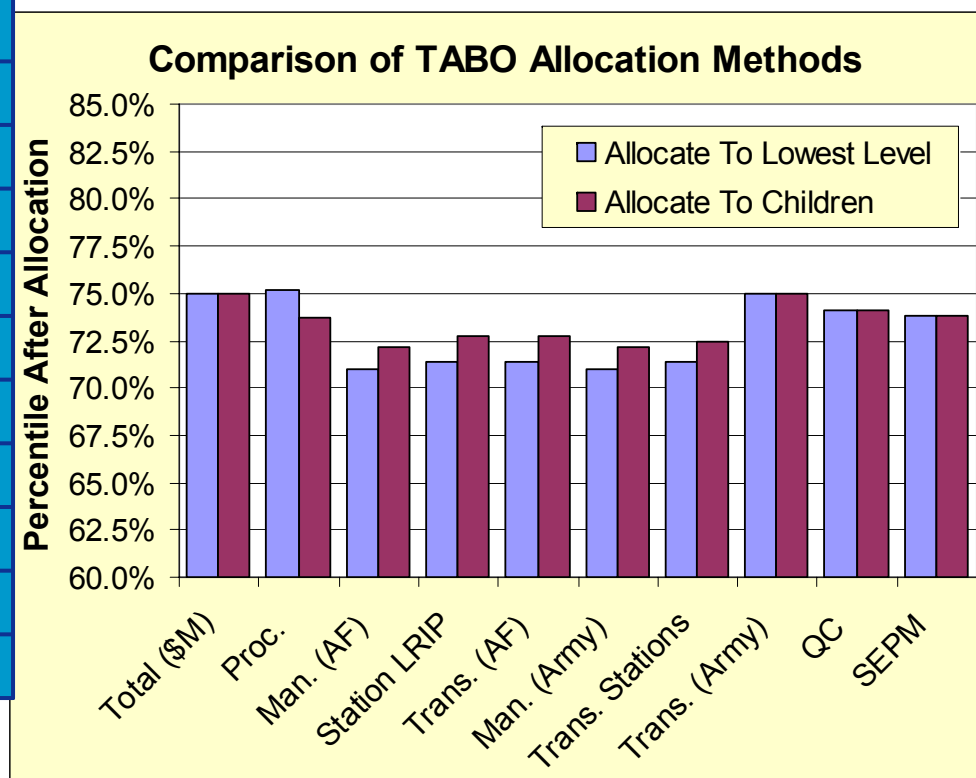


WBS/CES	75% -Tile	Std Dev	2 nd Level WBS Adjustment	Apply to 2 nd Level	%-Tiles	3 rd Level WBS Adjustment	Allocated	%-Tiles
Total (\$M)	\$608.9			\$608.9	75.0%		\$608.9	75.0%
Procurement	\$393.5	\$ 86.0	$-7.7 * 86 / 174 = -3.8$	\$389.7	73.7%		\$389.7	73.7%
Manufacturing (AF)	\$272.7	\$ 68.8		\$269.8	73.7%	$-8.9 * 69 / 99 = -6.2$	\$263.6	71.0%
Ground Station LRIP	\$0.88	\$ 0.3		\$0.87	73.7%	$-8.9 * 0.3 / 99 = -0.02$	\$0.85	71.4%
Transportation (AF)	\$2.00	\$ 0.6		\$1.97	73.7%	$-8.9 * 0.6 / 99 = -0.05$	\$1.92	71.4%
Manufacturing (Army)	\$125.9	\$ 29.6		\$124.5	73.7%	$-8.9 * 30 / 99 = -2.7$	\$121.8	71.0%
Transportable Stations	\$0.91	\$ 0.2		\$0.90	73.7%	$-8.9 * 0.2 / 99 = -0.02$	\$0.88	71.4%
Transportation (Army)	\$0.60	\$0.0		\$0.60			\$0.60	
Quality Control	\$10.8	\$ 4.3	$-7.7 * 4.3 / 174 = -0.2$	\$10.6	74.1%		\$10.6	74.1%
SEPM	\$212.3	\$ 84.1	$-7.7 * 84 / 174 = -3.7$	\$208.6	73.8%		\$208.6	73.8%
SUM OF CHILDREN (\$M)	\$616.6	\$174.4	Distribute \$-7.7	\$396.8	$\Sigma\sigma=99.5$	Distribute \$-8.9	\$608.9	75.0%

Cost Risk Allocation To Lowest WBS Level

Cost Risk Allocation To Immediate Children

Allocated	%-Tiles
\$608.9	75.0%
\$389.7	75.2%
\$263.6	71.0%
\$0.85	71.4%
\$1.92	71.4%
\$121.8	71.0%
\$0.88	71.4%
\$0.60	
\$10.6	74.1%
\$208.6	73.8%



Allocated	%-Tiles
\$608.94	75.0%
\$393.92	73.7%
\$266.43	72.2%
\$0.86	72.7%
\$1.95	72.7%
\$123.19	72.2%
\$0.89	72.5%
\$0.60	
\$10.39	74.1%
\$204.63	73.8%



Lather, Rinse, Repeat?

How Many Times Should We Iterate?

- **Once, usually; otherwise, twice**
 - Don't sweat the 0.001%-tile of confidence!
 - Too much precision is **misleading**...
 - If you allocate to the **penny**, it implies the estimate is **very precise**.
 - *Example: Guess the precision of these estimates: \$254,359.25 vs. \$250,000*
- **I *humbly* suggest that two values are essentially the same...**
 - ...at the 2nd significant figure of standard deviation or 1% of confidence
- **I *humbly* suggest that you round at the **second** digit of standard deviation***
 - *Example: Cost is \$2359.25, σ is \$238.77... thus, report \$2360 with σ of \$240*
 - Or round at **first digit** of the difference between values 1% confidence apart
 - *Example: Cost is \$268.36 @73%-tile and \$265.95 @ 72%-tile...*
 - *...difference is 2.38... thus, report cost @73%-tile as \$268*
 - Rounding at these positions retains an extra digit of padding for precision

Proverb

To err is human,
to measure it divine.

My Suggestion for Rounding

Round to the 2nd digit of the deviation after all intermediate calculations are complete.

* Examples at NIST Physical Constants web site^[2]



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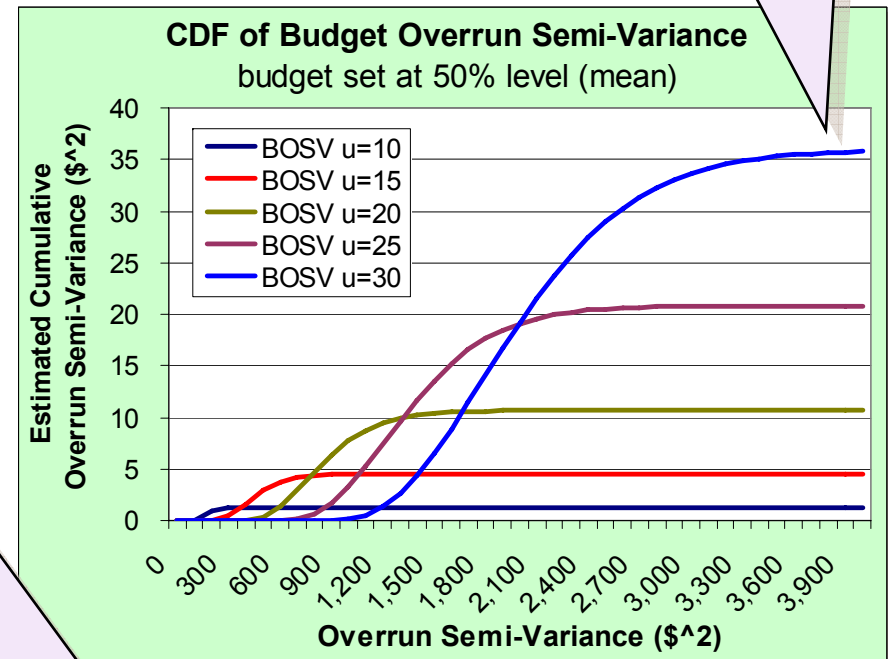
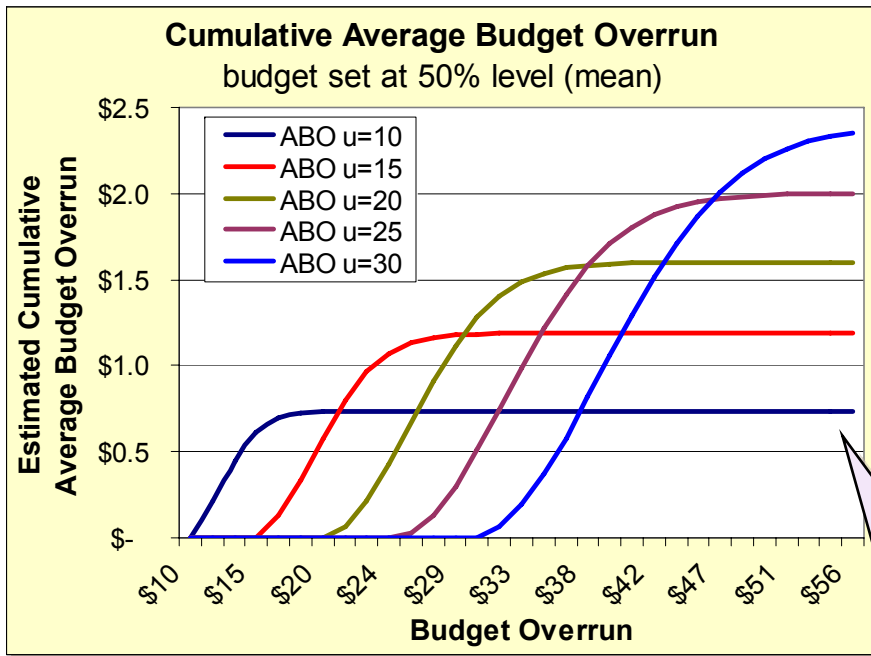
Minimizing Overrun Semi-Variance





- **Total Budget Overrun Semi-Variance Is *Tough to Visualize***
 - The charts below compare “Average” vs. “Semi-Variance”
 - “average overrun” progresses linearly as mean increases
 - “overrun variance” progresses at a rate of R^2 as overrun increases

3x the cost receives 9x the weighting



Heights represent element weightings

Mean	SD
\$10	\$3
\$15	\$5
\$20	\$6
\$25	\$8
\$30	\$9

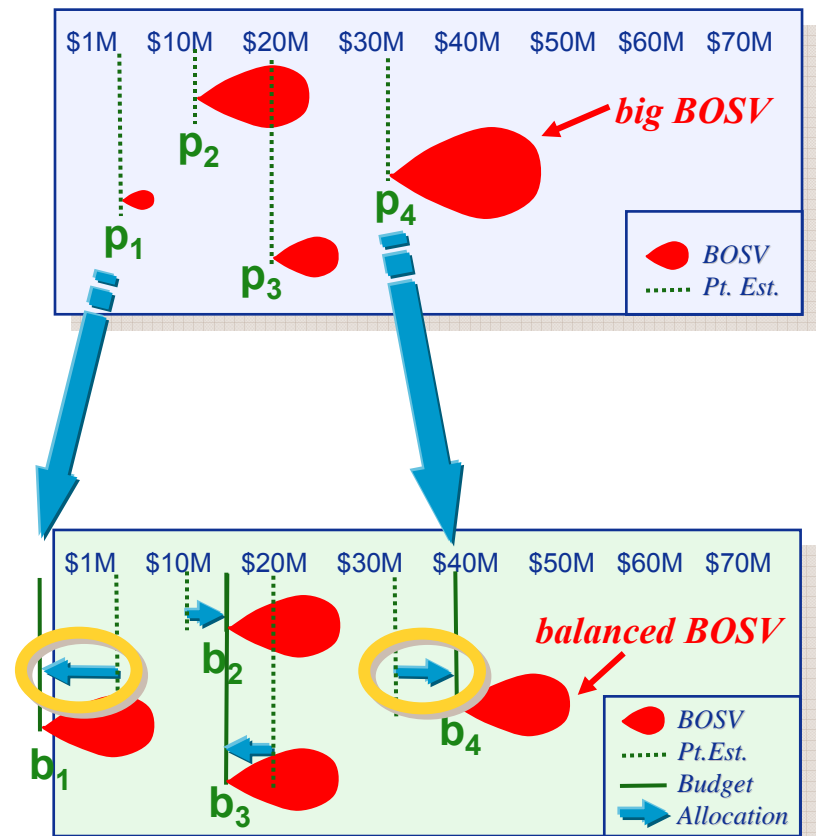
Mean	SD
\$10	\$3
\$15	\$5
\$20	\$6
\$25	\$8
\$30	\$9

Optimizing the “Variance Camp” Way...

- **Elements with large risk consequence have disproportionate importance**
 - Argument goes that we should protect them
 - Since we are dealing with variance, we must take correlation into account
- **Behavior of Optimal Solution**
 - A budget, B , such that the total budget overrun semi-variance (TBOSV), v_{total}^2 , is minimal.
 - All things being equal, we would want equal v_k
 - Desired BOSV decreases as element’s correlation to other elements increases
 - The method begins to resemble “cost camp” method as more and more correlations increase

Unfortunately...

- **The Optimal Solution is Not Viable**
 - Elements with small BOSV could move dramatically—potentially outside valid bounds
 - Tough math to solve, too.
- **How to Stay Within Distribution Bounds?**
 - Put limits on elements’ ranges of movement
 - Or, we can “anchor” our solution to something
 - Larger variances move further from “anchor”
 - Smaller variances remain near “anchor”





"Needs*" Somewhere To Start

■ "Allocating risk dollars back to WBS elements*" - a.k.a. the "Needs" Method

- Offers a scheme for the "Variance Camp" to reduce budget overrun semi-variance when allocating
- It uses PE as an "anchor" and distributes "risk dollars" to elements

NOT RECOMMENDED

$$b_k = pe_k + Risk\$ \sum_{i=1}^n \left(\frac{\rho_{ik} Need_i Need_k}{\sum_{j=1}^n \rho_{ij} Need_i Need_j} \right)$$

Where,

C is the probability level of the total budget

b_k is the allocated cost (budget line)

pe_k is the initial estimate for element k

$Risk\$ = F_T^{-1}(C) - pe_T$ is the total "at-risk" money to distribute

$Need_k = F_k^{-1}(C) - pe_k$ when $F_k(pe_k) < C$; otherwise $Need_k = 0$

ρ_{ij} is the correlation of elements i and j (full correlation matrix)

$F^{-1}(C)$ is inverse distribution function (returns cost at %-tile C)

Definition for
 $Need_k$ in Question

■ Issues with "Needs":

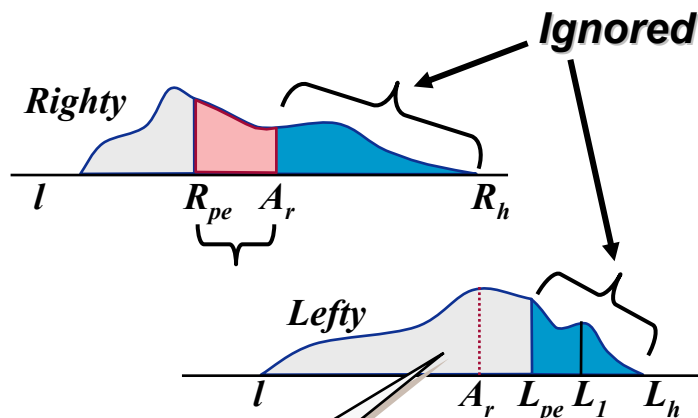
- Less risky (*left skew*) rows are subsidized, harming budgets for more risky (*right skew*) rows
- Undo burden to rows with $Need > 0$ which can potentially send small items below their 0%-tile
- $Need$ is analogous to semi-variance, yet an element's $Need$ changes when cost risk changes
- At higher cost risk, costs lose their "bolstering" from associations with elements with $Need=0$
- Method does not produce a solution at higher cost risk, when all elements' $Need=0$

■ The problem is with what is being minimized

* From "Allocating Risk Dollars Back to WBS Elements" Presentation, Stephen A. Book^[4]

"Needs" Is A Changin'

- Why is the equation " $Need_k = 0$ " so troublesome?
- In "Needs," only measures the range between PE and target percentile
 - Ignores cost risk above the target percentile (thus most of the budget's cost risk)
 - In fact, an element's measure of contribution could go away completely
- As long as our "anchor" doesn't move neither should our element's contribution



Proof that a row contributes to total's cost risk at any target probability level

"Risk" is defined as the distribution above PE

Let, Lefty + Righty = Total

Choose a percentile C such that $F(L_{pe}) > C$ and $F(R_{pe}) < C$

Since $F(L_{pe}) > C$, thus $Need_L = 0$

Assume, Lefty's cost risk does not contribute to total's cost risk above A_T

The total's cost risk is defined in the range $[A_T, L_h + R_h]$

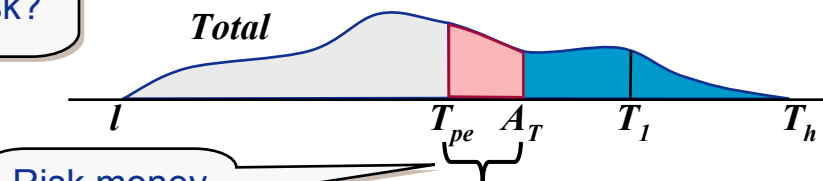
If $L_{pe} < L_h$, then some total value $T_1 > A_T$ exists where $T_1 = L_1 + R_h$, and $L_1 > L_{pe}$,

T_1 is part of total's cost risk, thus L_1 contributes to total's cost risk

However, a $Need = 0$ assumes the contribution to "risk" is 0

Therefore $Need_L$ must be > 0 whenever $L_{pe} < L_h$

Contributes
no cost risk?



Risk money
to allocate

Proverb ~~variance~~
The only constant is change.

The "New Needs" Method...

■ Apply Two Alterations

- Replace fluctuating *Need* with a **constant** measure, v
- Do not set *Need* (v) to zero

Proverb,
using
The ends justify the means.

■ Standard Deviation, σ , Is Weak Measure of v Since It Is a Symmetrical Measure*

- We want to estimate the BOSV, which lies to the **right** of the target budget
- σ **underestimates** the consequence for elements whose distributions are **skewed** to the right

■ For v^2 , I recommend Using **Positive Semi-Variance (PSV)**, σ_+^2

- It is a constant measure that takes distribution skew into account & offers a rough BOSV metric
- There are a number of ways to estimate PSV if you cannot calculate it directly

$$b_k = anchor_k + delta \frac{\sum_{j=1}^n \rho_{k,j} v_k v_j}{\sum_{i=1}^n \sum_{j=1}^n \rho_{i,j} v_i v_j}$$

$$v = \sigma_{+,k} = \sqrt{c_k \sum_{i=1}^s (x_i - \mu_k)^2}, \text{ for } x_i > \mu_k$$

where,

b_k is the allocated cost (budget line)
 $anchor_k$ is anchor point for element k
 $delta$ is the amount to distribute among elements
 $\sigma_{+,k}$ is the square root of the PSV
 ρ_{ij} is the (full) correlation of elements i and j

where,

$x_{i,k}$ is a point in element k 's random variable, X_k
 s is the number of data points in X
 c_k is the confidence level of the mean, μ_k

* Detailed in "Allocating Risk Dollars Back to WBS Elements" Presentation, Stephen A. Book^[4]



Six element example model with correlation*

Example Session						
WBS/CES	Pt. Est.	PE %-tile	Mean	Std Dev	95%-tile	Semi-Variance
Air Vehicle	\$333,396	15%	\$411,798	\$74,435	\$545,604	
Payload	\$11,416	14%	\$14,590	\$3,006	\$19,962	7,214,596
Propulsion	\$16,271	17%	\$20,496	\$4,499	\$28,744	17,007,376
Airframe	\$112,250	49%	\$116,277	\$26,776	\$165,003	593,555,769
Guidance	\$186,979	15%	\$251,304	\$61,745	\$366,670	3,327,328,489
IAT&C	\$6,480	9%	\$9,130	\$2,163	\$13,198	4,137,156

Correlation Matrix					
WBS/CES	Payload	Propulsion	Airframe	Guidance	I, A, T & C
Payload	1	0.32	0.32	0.21	0.33
Propulsion	0.32	1	0.25	0.17	0.15
Airframe	0.32	0.25	1	0.19	0.12
Guidance	0.21	0.17	0.19	1	0.18
I, A, T & C	0.33	0.15	0.12	0.18	1

We can estimate the semi-variance using a simple formula:

$$\sigma_+^2 \approx \frac{(high - \mu)^2}{4} \approx \frac{(cost_{95\%} - \mu)^2}{4}$$

More ways to estimate at end of presentation

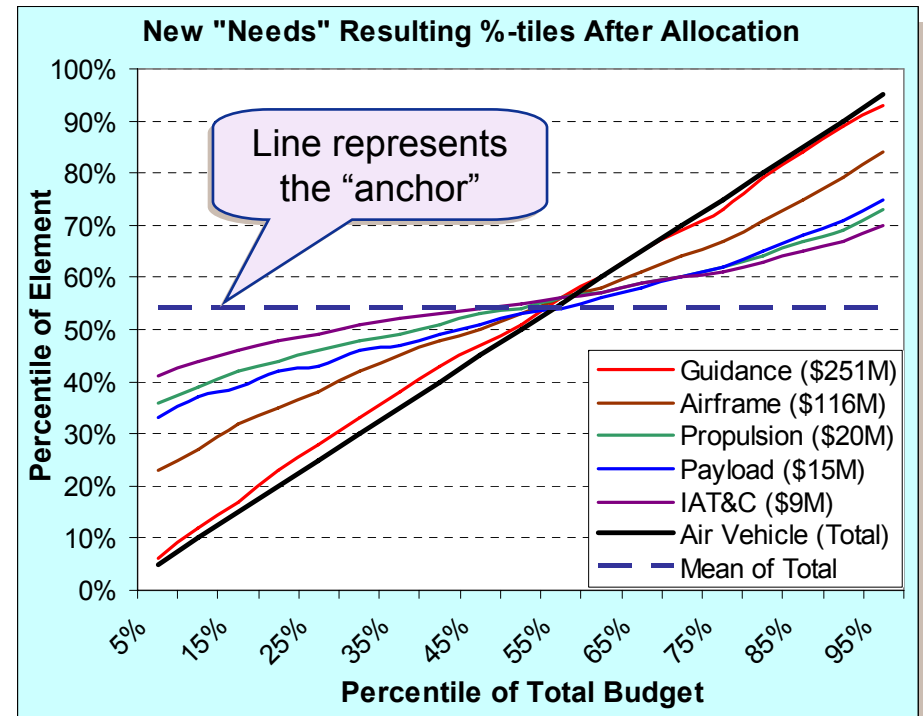
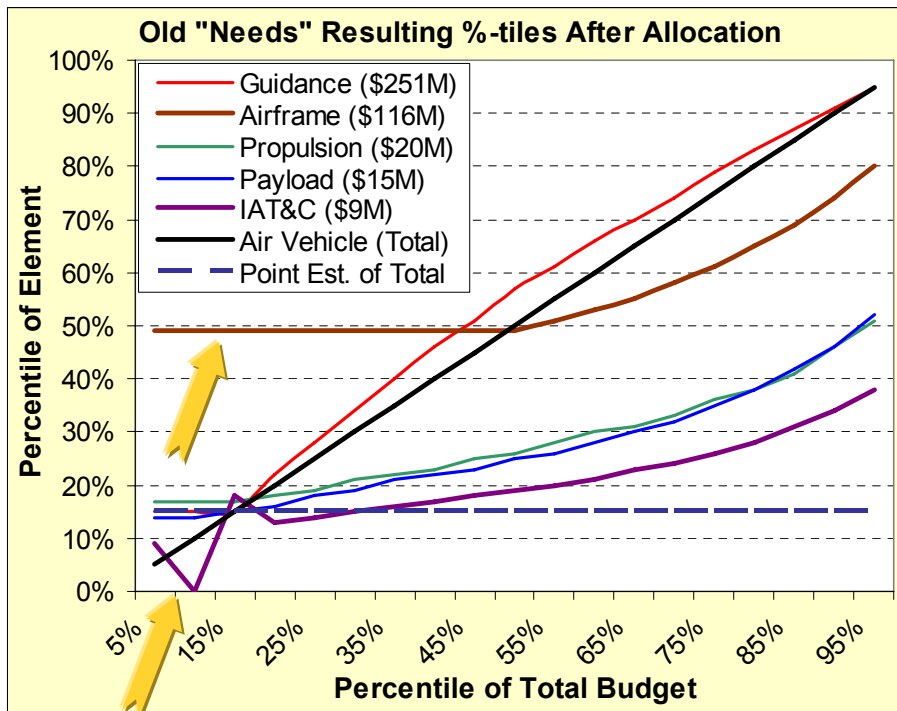
* "Air Vehicle Production Sub-WBS From AFCAA CRH Example^[5]

"Needs" Comparison - Mean

■ Comparison of Resulting Percentiles After Allocation Performed

- Chart shows "New Needs" offers more stability than "Old Needs"
- Anchoring at the mean offers "symmetry" for allocating at low and high percentiles

Scenario uses mean (μ) as anchor: $delta = budget_T - \mu_T$





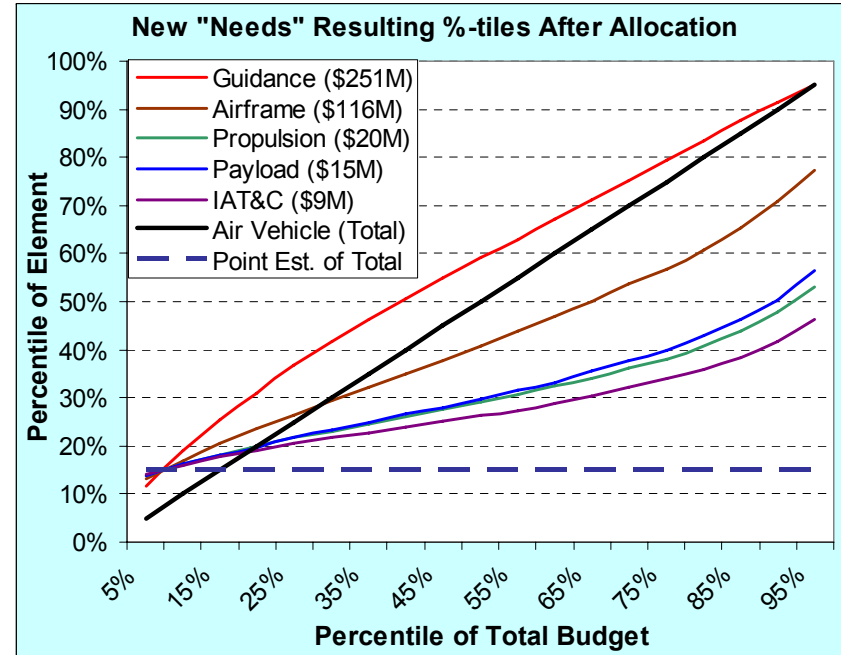
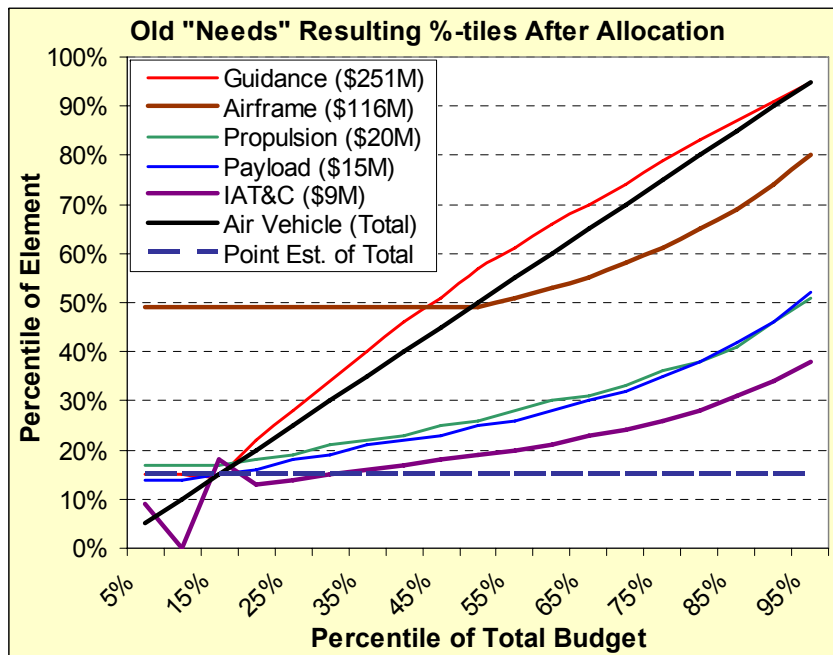
■ Alternate Comparison of Resulting Percentiles After Allocation

- "New Needs" supports alternatives to *anchor* and *delta* to suite your priorities
- Some prefer to use point estimate instead of mean. The total's pe_T is at 14%-tile.
- We find the $cost_i$ for each element at 14%-tile and use their sum them for to calc. *delta*

Scenario uses PE %-tile as anchor:

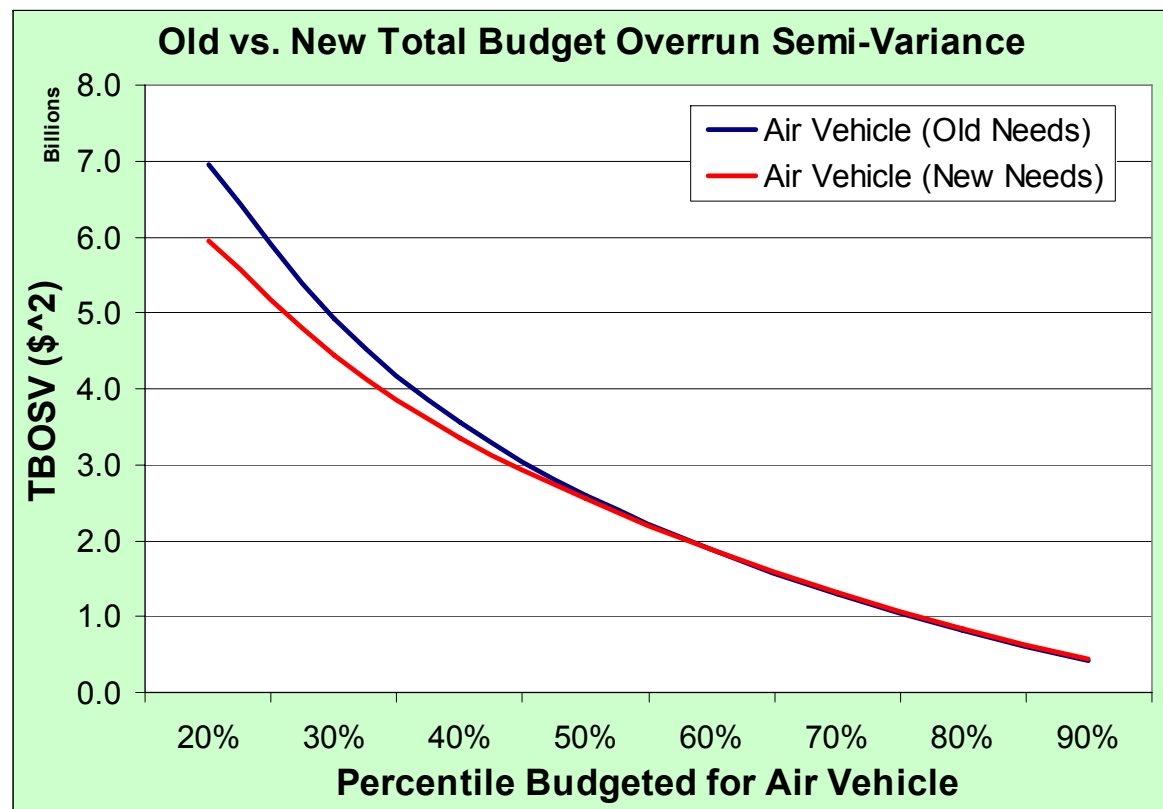
$$anchor_i = F_i^{-1}(F(pe_{total})) = [cost_i \text{ at } \% \text{ tile of } pe_{total}]$$

$$delta = budget_T - \sum_{i=1}^n anchor_i$$



The ultimate goal of the “Variance Camp” is to minimize TBOSV

- The chart below compares the TBOSV for “Old” and “New” methods
 - The “New Needs” method is using the P.E. %-tile scenario from previous slide.
- The new method outperforms the old for low confidences
- At high confidences, they are virtually identical





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Conclusion



- **Cost Risk allocation is a tool that serves a specific purpose**
 - Be sure that allocation serves your analysis goals
 - Only allocate when you have to encapsulate all money in WBS
 - Always allocate from where funds are managed
 - Allocate up or down the WBS
- **Two useful allocation methods were presented**
 - Consider the two camps of thought when picking a method
 - How to minimize the total average budget overrun (TABO)
 - How to (nearly) reduce the total budget overrun semi-variance (TBOSV)
 - Introduction to a more reliable “New Needs” method to replace old one
- **Round to stress the (lack of) precision of your numbers**
- **Be wary when discussing confidence levels after allocation**
 - This is a huge topic on its own!



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Questions?

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References:

[1] Information on the meanings of accuracy and precision:

http://en.wikipedia.org/wiki/Accuracy_and_precision

http://en.wikipedia.org/wiki/Significance_arithmetic#Uncertainty_and_error

[2] Examples of rounding at 2nd decimal of deviation:

<http://physics.nist.gov/cuu/Constants/>

Description of Semi-Variance

[3] “Selected Semi-Variance Estimators of Underreporting NonFarm Sole Proprietor Income,” Chih-Chin Ho, Internal Revenue Service, IRC1996_028

http://www.amstat.org/sections/srms/proceedings/papers/1996_028.pdf

Detailed description of the “Needs” method and model

[4] “Allocating Risk Dollars Back to WBS Elements” Stephen A. Book, Chief Technical Officer, MCR, LLC SSCAG/EACE/SCAF Meeting 19-21 September 2006, also presented at SCEA Conference June 2006, DoDCAS Symposium February 2007

Additional information on uncertainty analysis and time-phased cost risk allocation

[5] “AFCAA Cost Risk Handbook” Alfred Smith et. al., CR-1254-3, 9 April

[6] “Need’ Needs Kneading” John Sandberg, Tecolote Research, Inc., presented at SSCAG Meeting 17 Jan 2007



Here are general ways of calculating budget semi-variance, v^2

If you substitute μ for pe , you get the positive semi-variance, σ_+^2

General

General Form

$$v^2 = c \sum_{i=1}^s (x_i - pe)^2, \text{ for } x_i > pe$$

where, x is a point in X,
 t is the # of points in X, and c is the prob. of overrun

For Symmetrical Distribution Forms and $pe = \mu$

$$v^2 = \frac{\sigma^2}{2}$$

$pe = \mu$

Reference

Rough Estimates For Arbitrary Forms

$$v^2 = \frac{c}{4} (h - pe)^2$$

$$v^2 = \frac{c}{5} (h_{95\%} - pe)^2$$

For Triangular Distributions with $p \geq \text{Mode}$

$$v^2 = \frac{c}{6} (h - pe)^2$$

For Uniform Distributions

$$v^2 = \frac{c}{3} (h - pe)^2$$

For Triangular Distributions with $p < \text{Mode}$

$$v^2 = \frac{(m - pe)^4}{6(h - l)^2} + \frac{2(m - c)^3(c - l)}{3(h - l)^2} + \left(\frac{h - m}{h - l} \right) \left[\frac{(h - m)^2}{2} + \frac{4(m - c)(h - m)}{3} + (m - c)^2 \right]$$



Cumulative density for triangular

$$1 - c = F(x) = \begin{cases} 0, & x < l \\ \frac{(x-l)^2}{(\text{mode}-l)(h-l)}, & l \leq x < \text{mode} \\ 1 - \frac{(h-x)^2}{(h-\text{mode})(h-l)}, & \text{mode} \leq x \leq h \\ 1, & x > h \end{cases}$$

Inverse CDF for triangular

$$F^{-1}(p) = \begin{cases} l + \sqrt{p(\text{mode}-l)(h-l)}, & p < F(\text{mode}) \\ h - \sqrt{(1-p)(h-\text{mode})(h-l)}, & F(\text{mode}) \leq p \end{cases}$$

Probability density for triangular

$$f(x) = \begin{cases} 0, & x < l \\ \frac{2(x-l)}{(\text{mode}-l)(h-l)}, & l \leq x < \text{mode} \\ \frac{2(h-x)}{(h-\text{mode})(h-l)}, & \text{mode} \leq x \leq h \\ 0, & x > h \end{cases}$$

Mean and variance for triangular

$$\mu = \frac{l + \text{mode} + h}{3}$$

$$\sigma^2 = \frac{(\text{mode}-l)(\text{mode}-h) + (h-l)^2}{18}$$

Cumulative density for uniform

$$1 - c = F(x) = \begin{cases} 0, & x < l \\ \frac{(x-l)}{(h-l)}, & l \leq x \leq h \\ 1, & x > h \end{cases}$$

Inverse CDF for uniform

$$F^{-1}(p) = l + p(h-l)$$

Probability density for uniform

$$f(x) = \begin{cases} 0, & x < l \\ \frac{1}{(h-l)}, & l \leq x \leq h \\ 0, & x > h \end{cases}$$

Mean and variance for uniform

$$\mu = \frac{l+h}{2}$$

$$\sigma^2 = \frac{(h-l)^2}{12}$$



Derivations for Estimates

General Reference

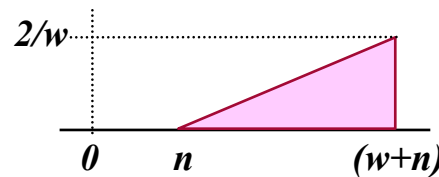
“cost-distance-squared”
of displaced triangle

$$v^2 = \int_0^w (x+n)^2 \left(\frac{2x}{w^2} \right) dx$$

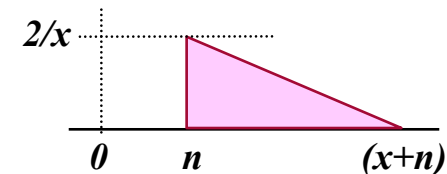
$$v^2 = \frac{2}{w^2} \int_0^w [x^3 + 2nx^2 + n^2x] dx$$

$$v^2 = \frac{2}{w^2} \left(\frac{w^4}{4} + \frac{2nw^3}{3} + \frac{n^2w^2}{2} \right)$$

$$v^2 = \frac{w^2}{2} + \frac{4nw}{3} + n^2$$



“cost-distance-squared”
of displaced triangle



$$v^2 = \int_0^w (x+n)^2 \left(\frac{2(w-x)}{w^2} \right) dx$$

$$v^2 = \frac{2}{w^2} \int_0^w [(x^2 + 2nx + n^2)(w-x)] dx$$

$$v^2 = \frac{2}{w^2} \int_0^w [wx^2 - x^3 + 2nwx - 2nx^2 + n^2w - n^2x] dx$$

$$v^2 = \frac{2}{w^3} \left(\frac{w^4}{3} - \frac{w^4}{4} + nw^3 - \frac{2nw^3}{3} + n^2w^2 - \frac{n^2w^2}{2} \right)$$

$$v = 2 \left[w^2 \left(\frac{1}{3} - \frac{1}{4} \right) + w \left(n - \frac{2n}{3} \right) + \left(n^2 - \frac{n^2}{2} \right) \right]$$

$$v^2 = \frac{w^2}{6} + \frac{2nw}{3} + n^2$$

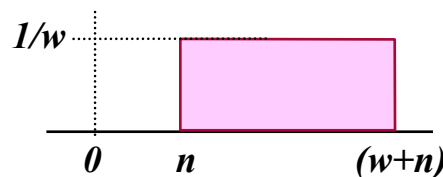
“cost-distance-squared”
of displaced rectangle

$$v^2 = \int_0^w (x+n)^2 \left(\frac{1}{w} \right) dx$$

$$v^2 = \frac{1}{w} \int_0^w (x^2 + 2nx + n^2) dx$$

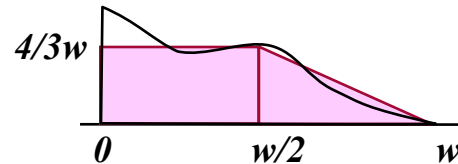
$$v^2 = \frac{1}{w} \left(\frac{1}{3} w^3 + nw^2 + n^2w \right)$$

$$v^2 = \frac{w^2}{3} + nw + n^2$$





Rough estimate for curve tapering down (fat tail)



$$v^2 = area_1 v_1^2 + area_2 v_2^2$$

$$v^2 = \frac{2}{3} \left(\frac{(w/2)^2}{3} \right) + \frac{1}{3} \left(\frac{(w/2)^2}{3} + (w/2)(w/2) + (w/2)^2 \right)$$

$$v^2 = w^2 \left(\frac{2}{4(9)} + \frac{1}{4(9)} + \frac{1}{12} + \frac{1}{12} \right) = w^2 \frac{9}{36}$$

$$v^2 = \frac{w^2}{4}$$

$$v^2 = area_1 v_1^2 + area_2 v_2^2 + area_3 v_3^2$$

$$v^2 = \frac{9}{13} \left(\frac{(w/3)^2}{6} \right) + \frac{3}{13} \left(\frac{(w/3)^2}{6} + \frac{2}{3} (w/3)(w/3) + (w/3)^2 \right) + \frac{1}{13} \left(\frac{(w/3)^2}{6} + \frac{2}{3} (w/3)(2w/3) + (2w/3)^2 \right)$$

$$v^2 = \frac{w^2}{13} \left(\frac{9(9)}{6(9)} + \frac{3}{6(9)} + \frac{6}{3(9)} + \frac{3}{9} + \frac{1}{6(9)} + \frac{4}{3(9)} + \frac{4}{9} \right) = w^2 \frac{147}{13(54)}$$

$$v^2 = \frac{49}{234} w^2 \approx \frac{w^2}{5}$$

Rough estimate for curve tapering up (thin tail)

