Stepwise Analysis in CO$TAT

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Agenda

- Background
- Challenges in explaining methodology
- Benefits of the Stepwise Analysis tool
- Example of CO$TAT Stepwise Analysis
- Advanced Topics
  - Handling multicollinearity
  - Missing data
  - Other advanced topics
- Developing best practices
- Questions

Background

Background and Environment

Assessing the needs of the organization:

- Lots of databases
- Many potential variables means multicollinear data
- May be incomplete data
- Statistical regressions are an acceptable methodology
- Repeatable results
- Decision-makers are informed about statistics

Background and Environment

Assessing the needs of the organization:

- Many first-uses of new CO$TAT tools
- Review CO$TAT results from a broad variety of organizations
- Informal best practices among teams
- Tend to be one or two CO$TAT specialists on a team (more technical)

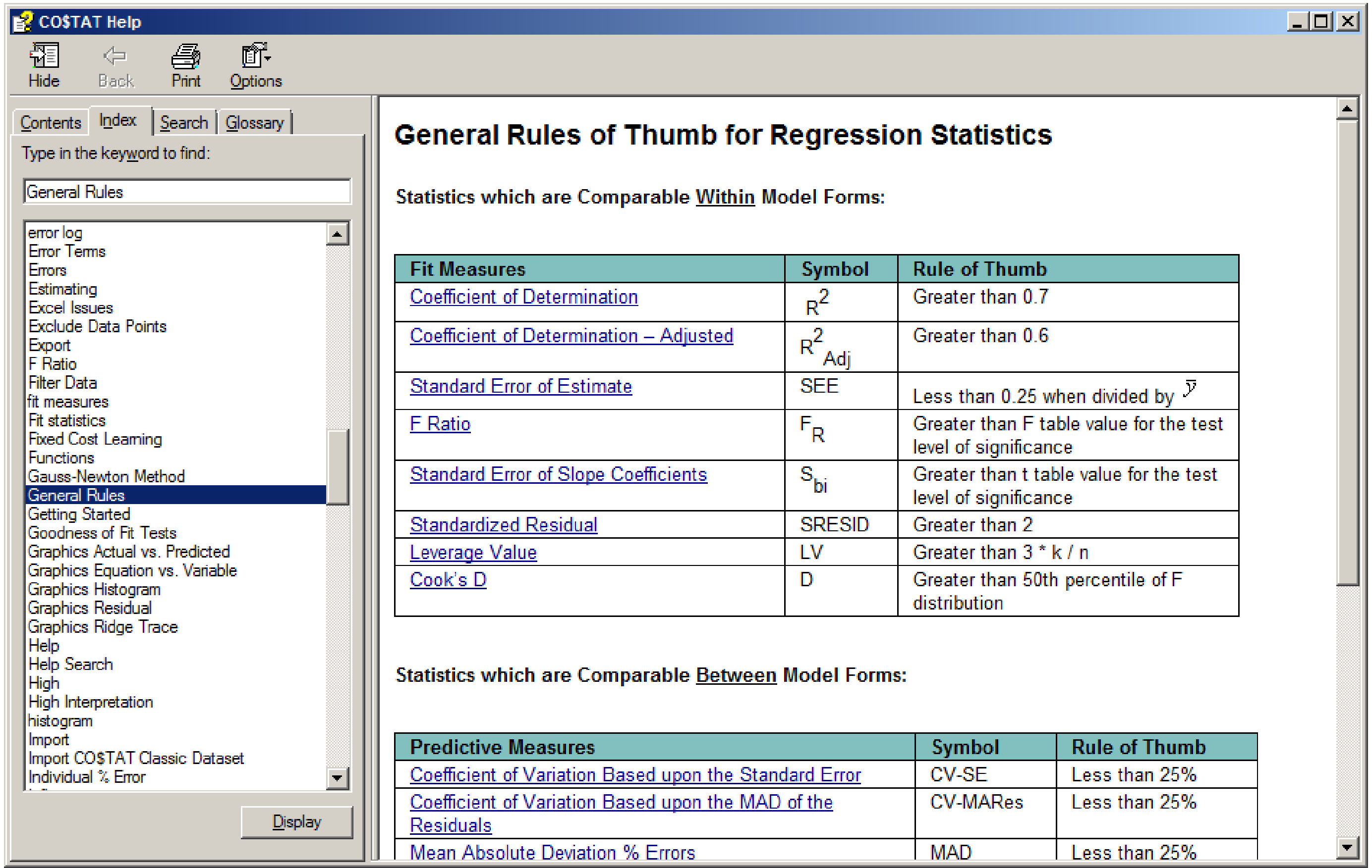
Challenges Explaining Methodology

Challenges Explaining Methodology (Lessons Learned)

- Decision Makers expect to be able to interpret the coefficients of the CER
- Must be intuitive
- Potentially many steps involved
- Non-linear progression adding variables
- Must be able to show why something didn't work
- Biases

Challenges Explaining Methodology (Guidance Challenges)

- Defense Acquisition University (DAU) recommends comparing all combinations of all variables
- CO$TAT Rules of Thumb



General Rules of Thumb for Regression Statistics

Statistics which are Comparable Within Model Forms:

Fit Measures	Symbol	Rule of Thumb
Coefficient of Determination	R^2	Greater than 0.7
Coefficient of Determination – Adjusted	R^2_{Adj}	Greater than 0.6
Standard Error of Estimate	SEE	Less than 0.25 when divided by $\bar{y}$
F Ratio	F_R	Greater than F table value for the test level of significance
Standard Error of Slope Coefficients	S_{bi}	Greater than t table value for the test level of significance
Standardized Residual	SRESID	Greater than 2
Leverage Value	LV	Greater than 3 * k / n
Cook's D	D	Greater than 50th percentile of F distribution

Statistics which are Comparable Between Model Forms:

Predictive Measures	Symbol	Rule of Thumb
Coefficient of Variation Based upon the Standard Error	CV-SE	Less than 25%
Coefficient of Variation Based upon the MAD of the Residuals	CV-MARes	Less than 25%
Mean Absolute Deviation % Errors	MAD	Less than 25%

Typical Documentation

CO$TAT has the Case utility. Can be exported to Excel.
Important documentation for showing range of coefficients.

Cases

Linear | Log Linear | Non Linear | Learning | Beta | Univariate | Distribution Finder | Analysis Summary

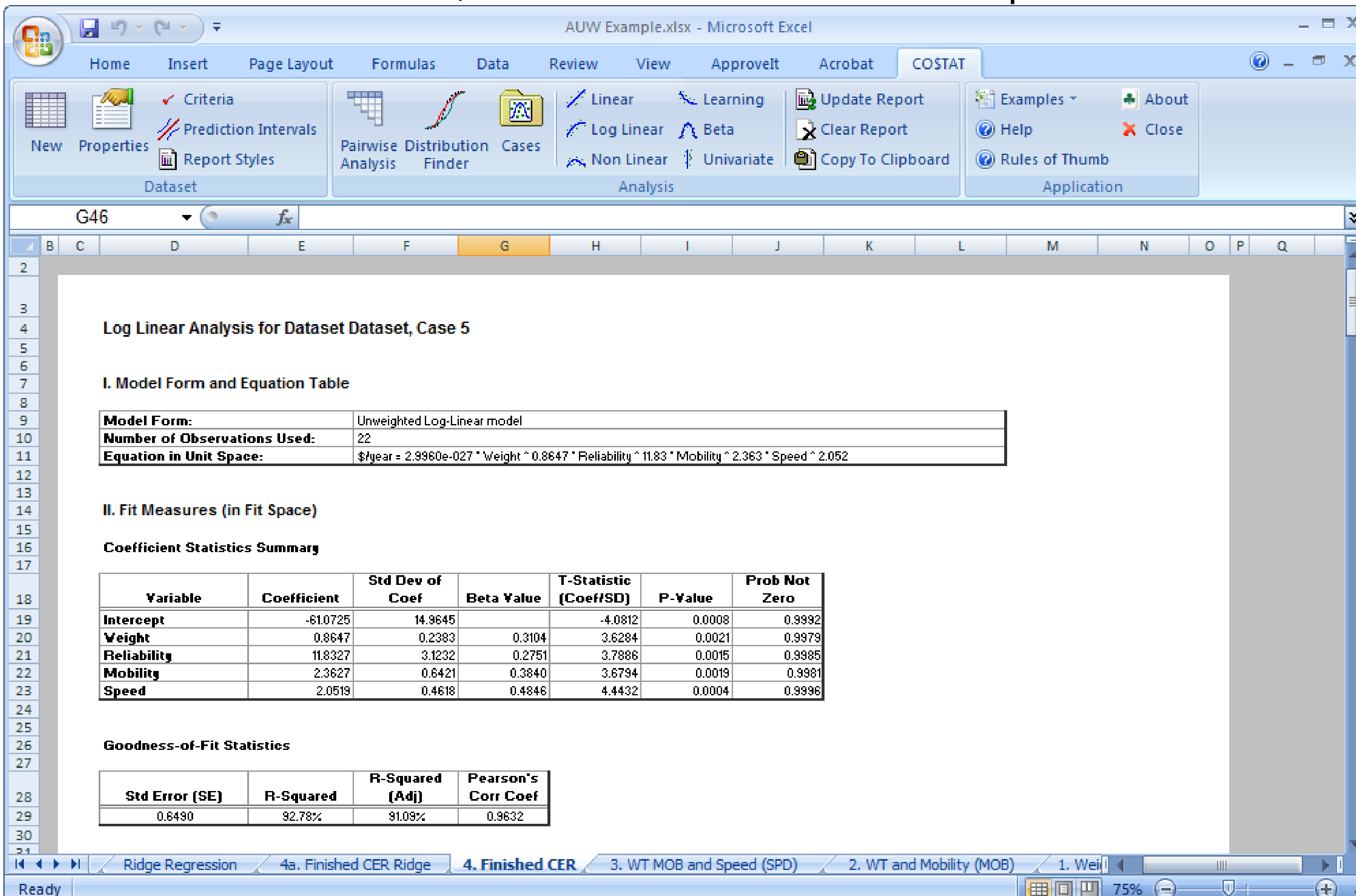
Calc	Type	Name	Status	Equation	DF	F-Prob	T-Prob Intercept	T-Prob b1	T-Prob b2	T-Prob b3	T-Prob b4	R² Adj(%)	SE (fit space) ▲1
☐	Log Linear	Case 125	Calculated	$/mile = 1.0376e-005 * var1P ^ 3.82	15	1.0000	0.9668	1.0000	0.9475	0.9996	1.0000	93.5826	0.5506
☐	Log Linear	Case 71	Calculated	$/mile = 0.0001523 * GVW ^ 0.5604	12	1.0000	0.9888	0.9296	0.9912	0.9999		90.6174	0.5576
☐	Log Linear	Case 63	Calculated	$/mile = 1.7483e-005 * GVW ^ 0.76	17	1.0000	0.9999	0.9976	0.9966	1.0000		92.1461	0.5759
☐	Log Linear	Case 64	Calculated	$/mile = 1.7483e-005 * GVW ^ 0.76	17	1.0000	0.9999	0.9976	0.9966	1.0000		92.1461	0.5759
☐	Log Linear	Case 61	Calculated	$/mile = 1.7149e-006 * GVW ^ 0.96	17	1.0000	1.0000	0.9997	0.9930	1.0000		92.9730	0.5768
☐	Log Linear	Case 94	Calculated	$/mile = 1.3445e-005 * GVW ^ 0.647	17	1.0000	1.0000	0.9928	0.9814	0.9319	0.9998	92.8923	0.5795
☐	Log Linear	Case 103	Calculated	$/mile = 6.8967e-007 * GVW ^ 0.65	17	1.0000	1.0000	0.9930	0.9794	0.9229	0.9999	92.8147	0.5826
☐	Log Linear	Case 77	Calculated	$/mile = 4.5910e-005 * GVW ^ 0.61	10	1.0000	0.9920	0.9270	0.6047	0.9636	0.9994	89.1117	0.5831
☐	Log Linear	Case 102	Calculated	$/mile = 3.6206e-008 * GVW ^ 0.657	17	1.0000	1.0000	0.9932	0.9782	0.9216	0.9999	92.7728	0.5843
☐	Log Linear	Case 35	Calculated	Cost_mile = 1.9510e-007 * GVW ^ 1.	13	1.0000	0.9996	1.0000				72.2508	0.5873
☐	Log Linear	Case 39	Calculated	Cost_mile = 1.9510e-007 * GVW ^ 1.	13	1.0000	0.9996	1.0000				72.2508	0.5873
☐	Log Linear	Case 46	Calculated	Cost_mile = 1.9510e-007 * GVW ^ 1.	13	1.0000	0.9996	1.0000				72.2508	0.5873
☐	Log Linear	Case 105	Calculated	$/mile = 1.9510e-007 * GVW ^ 1.81	13	1.0000	0.9996	1.0000				72.2508	0.5873
☐	Log Linear	Case 106	Calculated	$/mile = 1.9510e-007 * GVW ^ 1.81	13	1.0000	0.9996	1.0000				72.2508	0.5873
☐	Log Linear	Case 122	Calculated	$/mile = 6.8457e-008 * var1P ^ 3.88	17	1.0000	1.0000	1.0000	0.9997	1.0000	0.9817	92.5298	0.5941
☐	Log Linear	Case 59	Calculated	$/mile = 2.5306e-006 * GVW ^ 0.924	16	1.0000	1.0000	0.9993	0.8538	0.9115	0.9988	92.4363	0.5978
☐	Log Linear	Case 51	Calculated	Cost_mile = 1.8585e-007 * GVW ^ 2.	12	0.9997	0.9994	0.9457	0.3597			70.5033	0.6055
☐	Log Linear	Case 92	Calculated	$/mile = 2.7054e-005 * CW ^ 0.7995	16	1.0000	1.0000	0.9977	0.8695	0.7509	0.9611	92.0897	0.6113
☐	Log Linear	Case 24	Calculated	Cost_mile = 9.8518e-008 * GVW ^ 1.	12	0.9998	0.9995	0.9999	0.5230			71.3532	0.6173
☐	Log Linear	Case 87	Calculated	$/mile = 0.0001176 * GVW ^ 0.8602	17	1.0000	0.9990	0.9985	0.9994	0.9997	0.9993	91.9288	0.6175
☐	Log Linear	Case 127	Calculated	$/mile = 0.0002267 * GVW ^ 0.811 *	16	1.0000	0.9969	0.9966	0.9989	0.9965	0.6051	91.8152	0.6218
☐	Log Linear	Case 128	Calculated	$/mile = 4.2187e-005 * GVW ^ 0.811	16	1.0000	0.9995	0.9966	0.9989	0.6051	0.9992	91.8152	0.6218
☐	Log Linear	Case 89	Calculated	$/mile = 1.9720e-005 * GVW ^ 0.87	17	1.0000	0.9999	0.9984	0.8805	0.9971	0.9988	91.4183	0.6367
☐	Log Linear	Case 99	Calculated	$/mile = 1.9720e-005 * GVW ^ 0.87	17	1.0000	0.9999	0.9984	0.8805	0.9971	0.9988	91.4183	0.6367
☐	Log Linear	Case 100	Calculated	$/mile = 1.9720e-005 * GVW ^ 0.87	17	1.0000	0.9999	0.9984	0.8805	0.9971	0.9988	91.4183	0.6367
☐	Log Linear	Case 101	Calculated	$/mile = 1.9720e-005 * GVW ^ 0.87	17	1.0000	0.9999	0.9984	0.8805	0.9971	0.9988	91.4183	0.6367
☐	Log Linear	Case 22	Calculated	Cost_mile = 2.1636e-007 * Annual_F	11	0.9996	0.9987	0.1876	0.9991	0.0804		67.3290	0.6379
☐	Log Linear	Case 9	Calculated	Cost_mile = 1.5363e-009 * GVW ^ 3.	4	0.8838	0.9412	0.8981	0.4501	0.5023		54.2086	0.6479

Criteria: None

OK | Cancel | Apply | Help

Typical Documentation

The documentation exported to ACEIT (shown below) or JIAT is shown here. As a stand-alone document, the file needs some further explanation.



Log Linear Analysis for Dataset Dataset, Case 5

I. Model Form and Equation Table

Model Form:	Unweighted Log-Linear model
Number of Observations Used:	22
Equation in Unit Space:	$/year = 2.9960e-027 * Weight ^ 0.8647 * Reliability ^ 11.83 * Mobility ^ 2.363 * Speed ^ 2.052

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

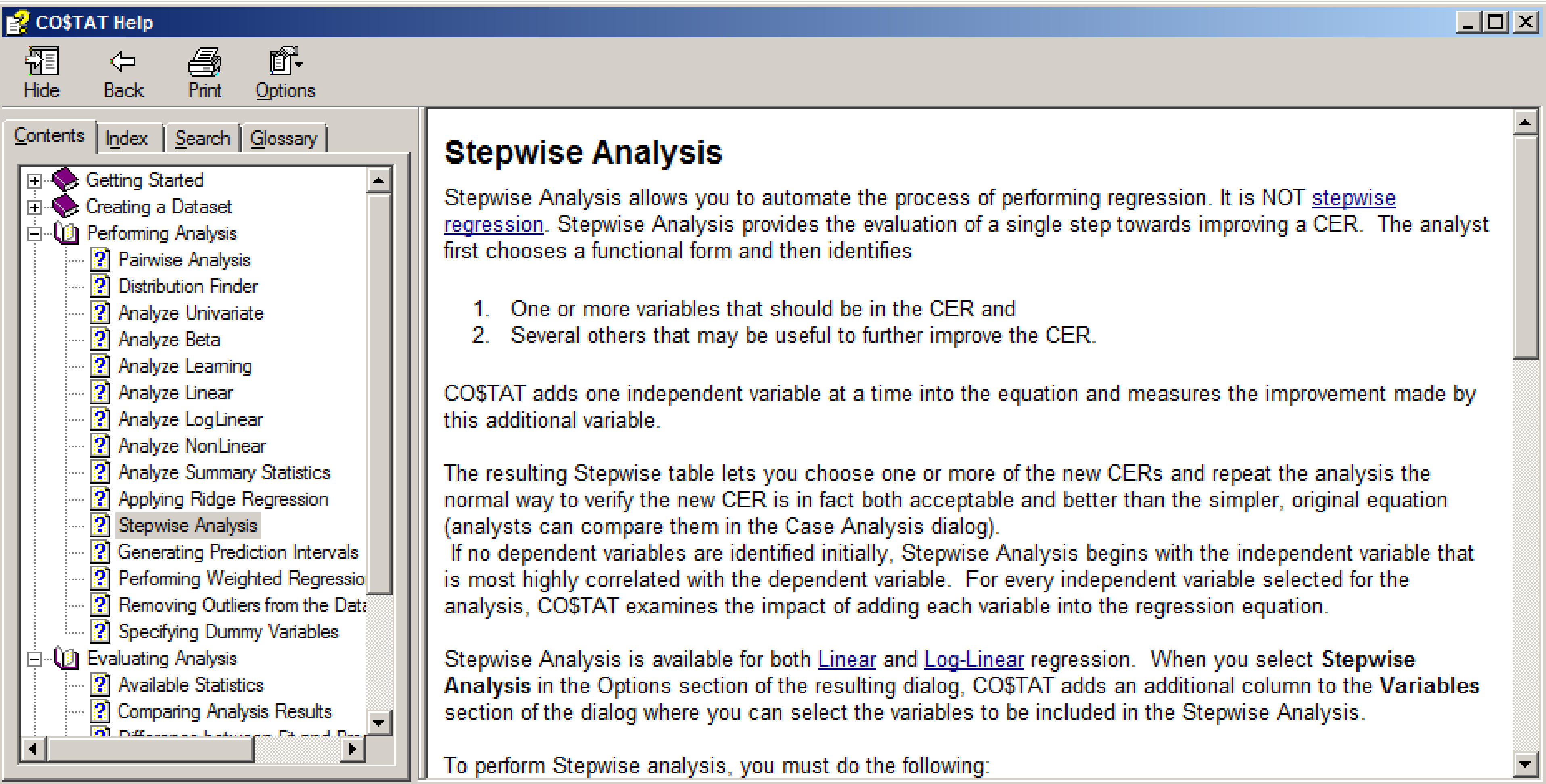
Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	-61.0725	14.9645		-4.0812	0.0008	0.9992
Weight	0.8647	0.2383	0.3104	3.6284	0.0021	0.9979
Reliability	11.8327	3.1232	0.2751	3.7886	0.0015	0.9985
Mobility	2.3627	0.6421	0.3840	3.6794	0.0019	0.9981
Speed	2.0519	0.4618	0.4846	4.4432	0.0004	0.9996

Goodness-of-Fit Statistics

Std Error (SE)	R-Squared	R-Squared (Adj)	Pearson's Corr Coef
0.6490	92.78%	91.09%	0.9632

Benefits of the Stepwise Analysis Tool

Benefits of the Stepwise Analysis Tool

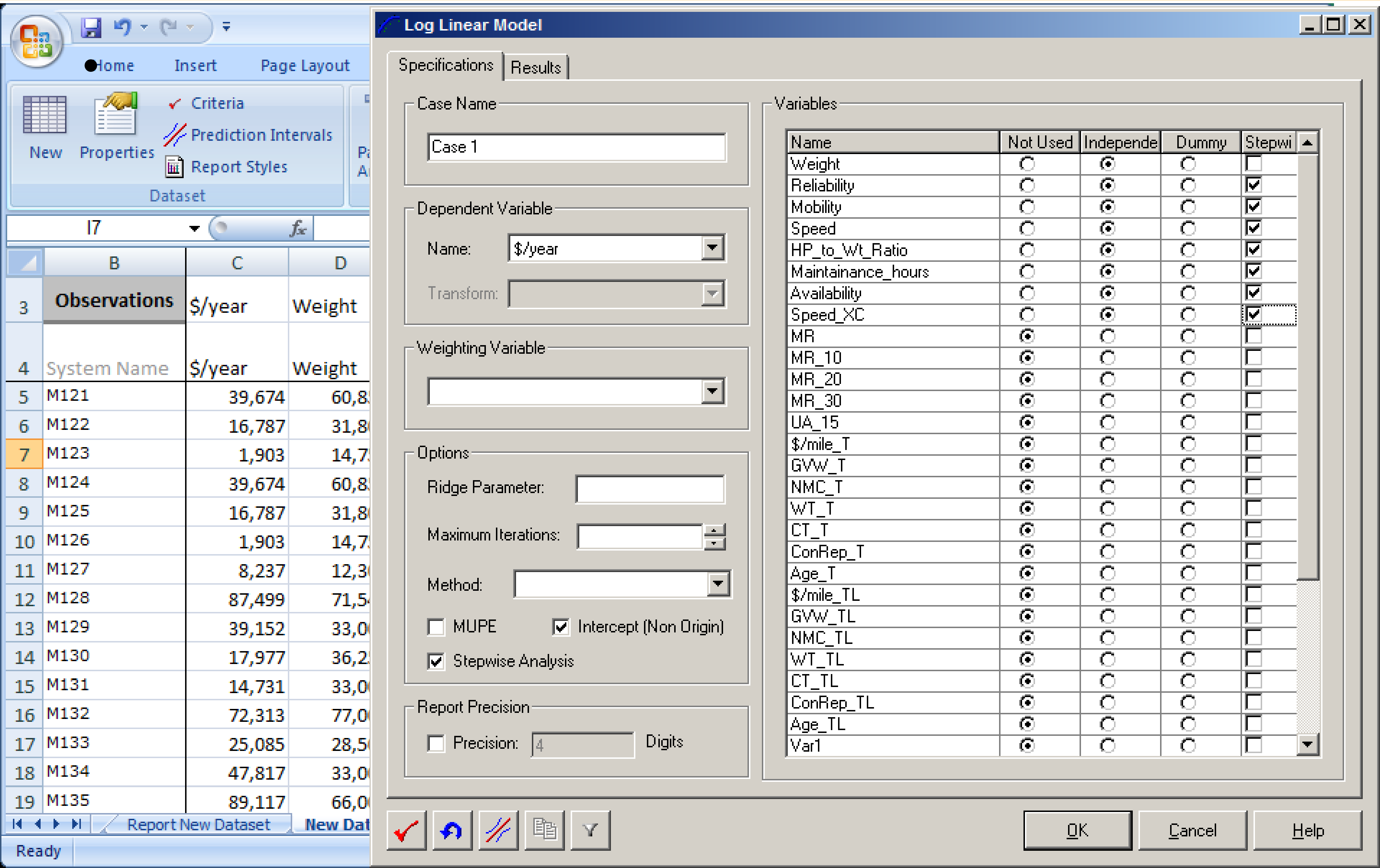


Benefits of the Stepwise Analysis Tool

- Quicker way to generate results
- Can compare several cases at once
- More compact way to show steps
- Facilitates producing all the documentation in one file with multiple tabs
- Ability to quickly compare transformations

Example of Stepwise Analysis

Example of CO$TAT Stepwise Analysis



Example of CO$TAT Stepwise Analysis

Log Linear Analysis for Dataset Dataset, Case 1

I. Model Form and Equation Table

Model Form:	Unweighted Log-Linear model
Number of Observations Used:	22
Equation in Unit Space:	$/year = 5.7043e-006 * Weight ^ 2.043

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	-12.0743	4.3027		-2.8062	0.0109	0.9891
Weight	2.0428	0.4233	0.7335	4.8256	0.0001	0.9999

Goodness-of-Fit Statistics

Std Error (SE)	R-Squared	R-Squared (Adj)	Pearson's Corr Coef
1.5139	53.80%	51.49%	0.7335

Analysis of Variance

Due To	DF	Sum of Sqr (SS)	Mean SQ = SS/DF	F-Stat	P-Value	Prob Not Zero
Regression	1	53.3727	53.3727	23.2861	0.0001	0.9999
Residual (Error)	20	45.8408	2.2920			
Total	21	99.2135				

Outlier Analysis Summary

Example of CO$TAT Stepwise Analysis

Here I will choose to add Mobility

D	E	F	G	H	I	J	K
19	5.7854	8.8911	-3.1057	0.3273	-2.1011	0.0467	0.1082
20	6.5548	5.7143	0.8404	0.6851	0.6225	0.2048	0.0499
21	5.8162	5.3251	0.4911	0.7571	0.3746	0.2501	0.0234
22	4.2970	5.6104	-1.3134	0.7041	-0.9800	0.2163	0.1325

SE = 1.5139, Mean = 8.6301, Coef. of Var. = 17.54% in Fit Space

R denotes an observation with an unusual Dependent variable value.

Partial Correlation Coefficients of Independent Stepwise Variables

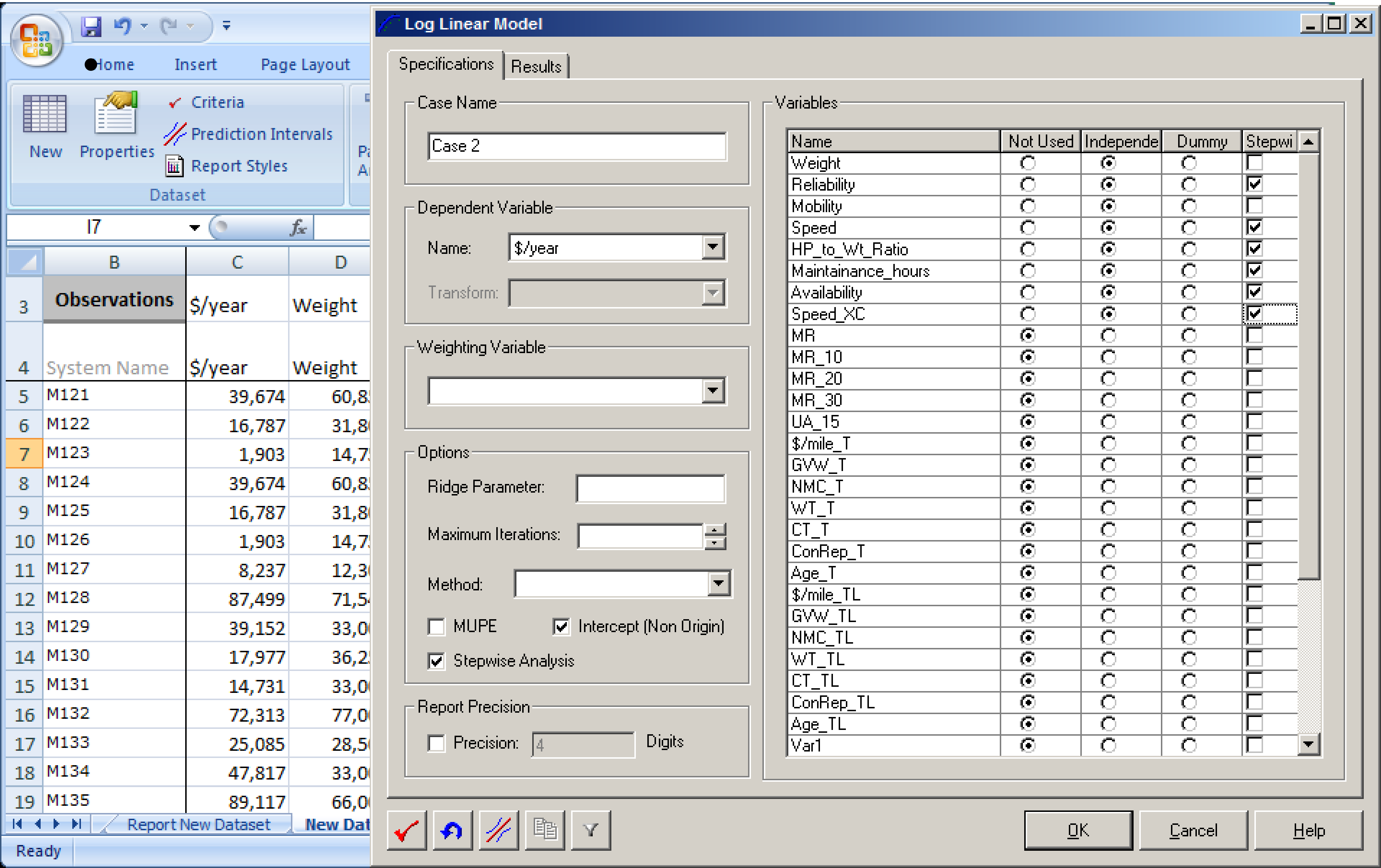
Ind Variable	Partial Correls	T-Statistic	P-Value	Equation	R-Squared (Adj)	MAD (%Error)
Original Equation				$/year = 5.7043e-006 * Weight ^ 2.043	51.49%	336.93%
Mobility	0.7712	5.2802	0.0000	$/year = 0.06526 * Weight ^ 0.9716 * Mobility ^ 4	79.30%	88.84%
Speed	0.7515	4.9654	0.0000	$/year = 3.4511e-005 * Weight ^ 0.9649 * Speed ^ 2.714	77.77%	160.92%
Speed_XC	0.4131	1.9773	0.0626	$/year = 2.9857e-005 * Weight ^ 1.453 * Speed_XC ^ 1.589	57.65%	274.91%
Availability	0.2280	1.0206	0.3202	$/year = 1.7980e-006 * Weight ^ 2.235 * Availability ^ 13.95	51.59%	294.71%
HP_to_Wt_Ratio	0.2187	0.9770	0.3408	$/year = 1.5671e-008 * Weight ^ 2.017 * HP_to_Wt_Ratio ^ 2.152	51.37%	304.62%
Reliability	0.1730	0.7658	0.4532	$/year = 4.6823e-017 * Weight ^ 2.166 * Reliability ^ 5.405	50.46%	307.36%
Maintainance_hours	-0.5251	-2.6896	0.0145	$/year = 0.006409 * Weight ^ 1.426 * Maintainance_hours ^ (-0.6257)	63.01%	196.09%

III. Predictive Measures (in Unit Space)

Percentage Error Table

Obs #	Name	Actuals	Predicted	Residuals	% Errors	Flags
1	M121	39673.5759	33856.3568	5817.2192	-14.6627	
2	M122	16787.0458	8997.2824	7789.7634	-46.4034	

Example of CO$TAT Stepwise Analysis



Example of CO$TAT Stepwise Analysis

Log Linear Analysis for Dataset Dataset, Case 2

I. Model Form and Equation Table

Model Form:	Unweighted Log-Linear model
Number of Observations Used:	22
Equation in Unit Space:	$/year = 0.06526 * Weight ^ 0.9716 * Mobility ^ 4

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	-2.7294	3.3212		-0.8218	0.4213	0.5787
Weight	0.9716	0.3430	0.3488	2.8330	0.0106	0.9894
Mobility	4.0004	0.7576	0.6502	5.2802	0.0000	1.0000

Goodness-of-Fit Statistics

Std Error (SE)	R-Squared	R-Squared (Adj)	Pearson's Corr Coef
0.9889	81.27%	79.30%	0.9015

Analysis of Variance

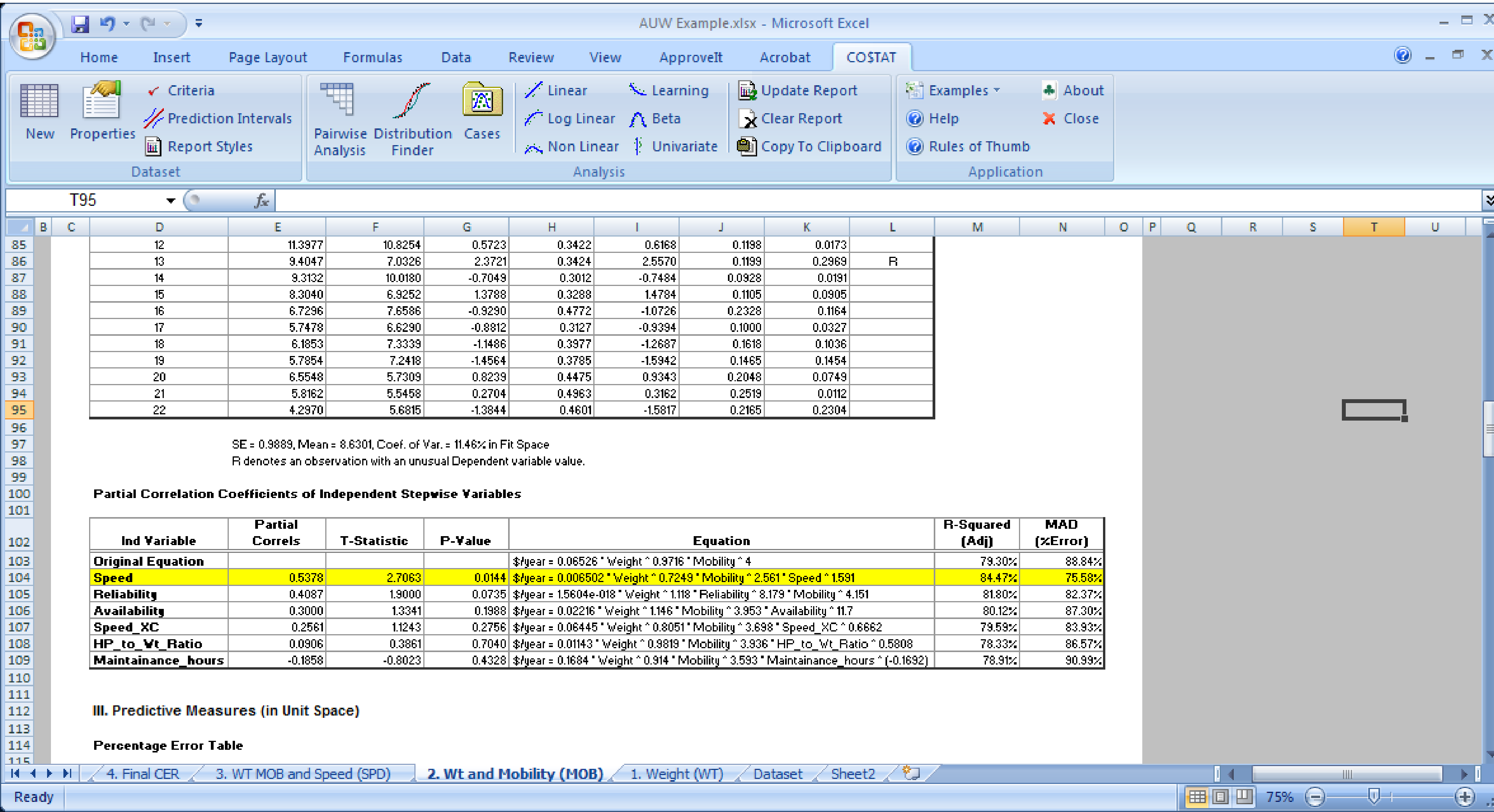
Due To	DF	Sum of Sqr (SS)	Mean SQ = SS/DF	F-Stat	P-Value	Prob Not Zero
Regression	2	80.6347	40.3174	41.2314	0.0000	1.0000
Residual (Error)	19	18.5788	0.9778			
Total	21	99.2135				

Further Analysis of Variance

(SS explained by each variable when entered in the order given)

Example of CO$TAT Stepwise Analysis

Next I might add speed



D	E	F	G	H	I	J	K	L
12	11.3977	10.8254	0.5723	0.3422	0.6168	0.1198	0.0173	
13	9.4047	7.0326	2.3721	0.3424	2.5570	0.1199	0.2969	R
14	9.3132	10.0180	-0.7049	0.3012	-0.7484	0.0928	0.0191	
15	8.3040	6.9252	1.3788	0.3288	1.4784	0.1105	0.0905	
16	6.7296	7.6586	-0.9290	0.4772	-1.0726	0.2328	0.1164	
17	5.7478	6.6290	-0.8812	0.3127	-0.9394	0.1000	0.0327	
18	6.1853	7.3339	-1.1486	0.3977	-1.2687	0.1618	0.1036	
19	5.7854	7.2418	-1.4564	0.3785	-1.5942	0.1465	0.1454	
20	6.5548	5.7309	0.8239	0.4475	0.9343	0.2048	0.0749	
21	5.8162	5.5458	0.2704	0.4963	0.3162	0.2519	0.0112	
22	4.2970	5.6815	-1.3844	0.4601	-1.5817	0.2165	0.2304	

SE = 0.9889, Mean = 8.6301, Coef. of Var. = 11.46% in Fit Space
R denotes an observation with an unusual Dependent variable value.

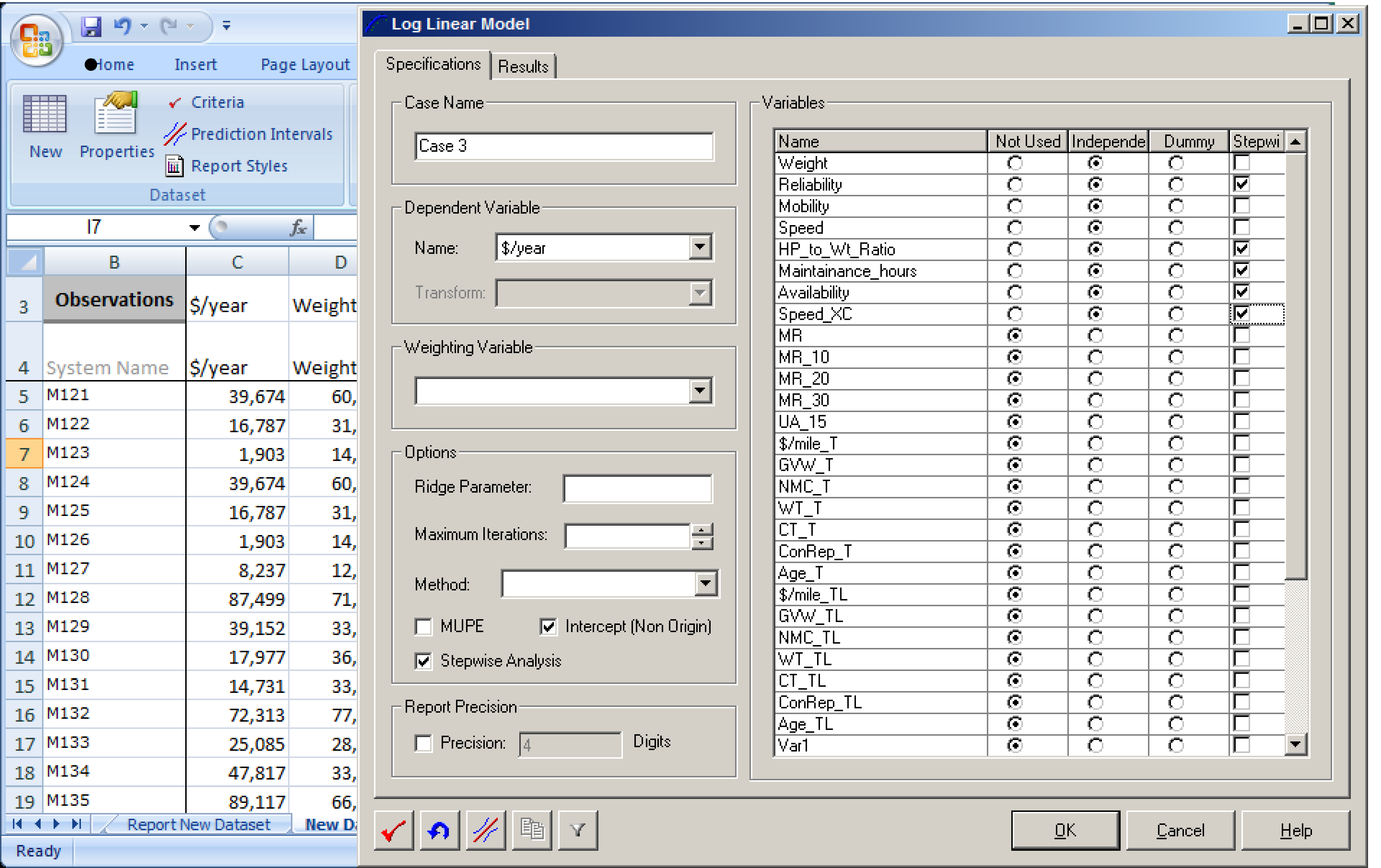
Partial Correlation Coefficients of Independent Stepwise Variables

Ind Variable	Partial Correls	T-Statistic	P-Value	Equation	R-Squared (Adj)	MAD (%Error)
Original Equation				$/year = 0.06526 * Weight ^ 0.9716 * Mobility ^ 4	79.30%	88.84%
Speed	0.5378	2.7063	0.0144	$/year = 0.006502 * Weight ^ 0.7249 * Mobility ^ 2.561 * Speed ^ 1.591	84.47%	75.58%
Reliability	0.4087	1.9000	0.0735	$/year = 1.5604e-018 * Weight ^ 1.118 * Reliability ^ 8.179 * Mobility ^ 4.151	81.80%	82.37%
Availability	0.3000	1.3341	0.1988	$/year = 0.02216 * Weight ^ 1.146 * Mobility ^ 3.953 * Availability ^ 11.7	80.12%	87.30%
Speed_XC	0.2561	1.1243	0.2756	$/year = 0.06445 * Weight ^ 0.8051 * Mobility ^ 3.698 * Speed_XC ^ 0.6662	79.59%	83.93%
HP_to_Wt_Ratio	0.0906	0.3861	0.7040	$/year = 0.01143 * Weight ^ 0.9819 * Mobility ^ 3.936 * HP_to_Wt_Ratio ^ 0.5808	78.33%	86.57%
Maintainance_hours	-0.1858	-0.8023	0.4328	$/year = 0.1684 * Weight ^ 0.914 * Mobility ^ 3.593 * Maintainance_hours ^ (-0.1692)	78.91%	90.99%

III. Predictive Measures (in Unit Space)

Percentage Error Table

Example of CO$TAT Stepwise Analysis



Example of CO$TAT Stepwise Analysis

AUW Example.xlsx - Microsoft Excel

Log Linear Analysis for Dataset Dataset, Case 3

I. Model Form and Equation Table

Model Form:	Unweighted Log-Linear model
Number of Observations Used:	22
Equation in Unit Space:	$/year = 0.006502 * Weight ^ 0.7249 * Mobility ^ 2.561 * Speed ^ 1.591

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	-5.0357	3.0003		-1.6784	0.1104	0.8896
Weight	0.7249	0.3107	0.2603	2.3330	0.0314	0.9686
Mobility	2.5613	0.8447	0.4163	3.0323	0.0071	0.9929
Speed	1.5912	0.5880	0.3758	2.7063	0.0144	0.9856

Goodness-of-Fit Statistics

Std Error (SE)	R-Squared	R-Squared (Adj)	Pearson's Corr Coef
0.8565	86.69%	84.47%	0.9311

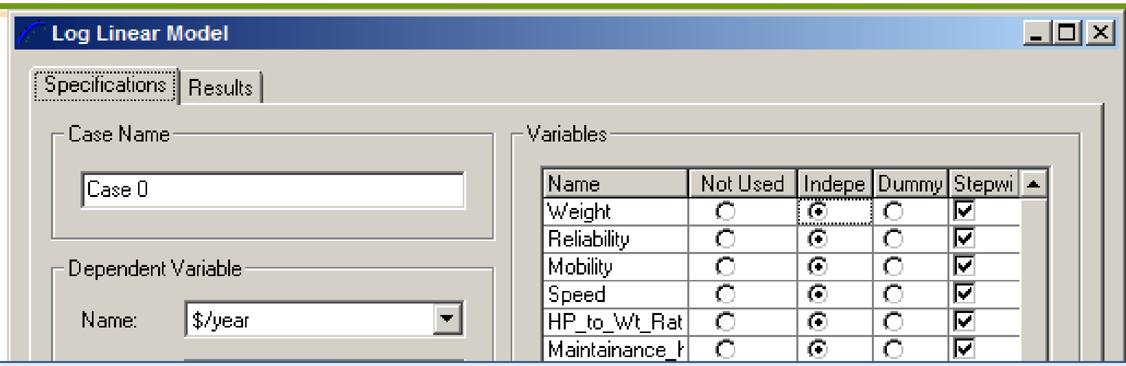
Analysis of Variance

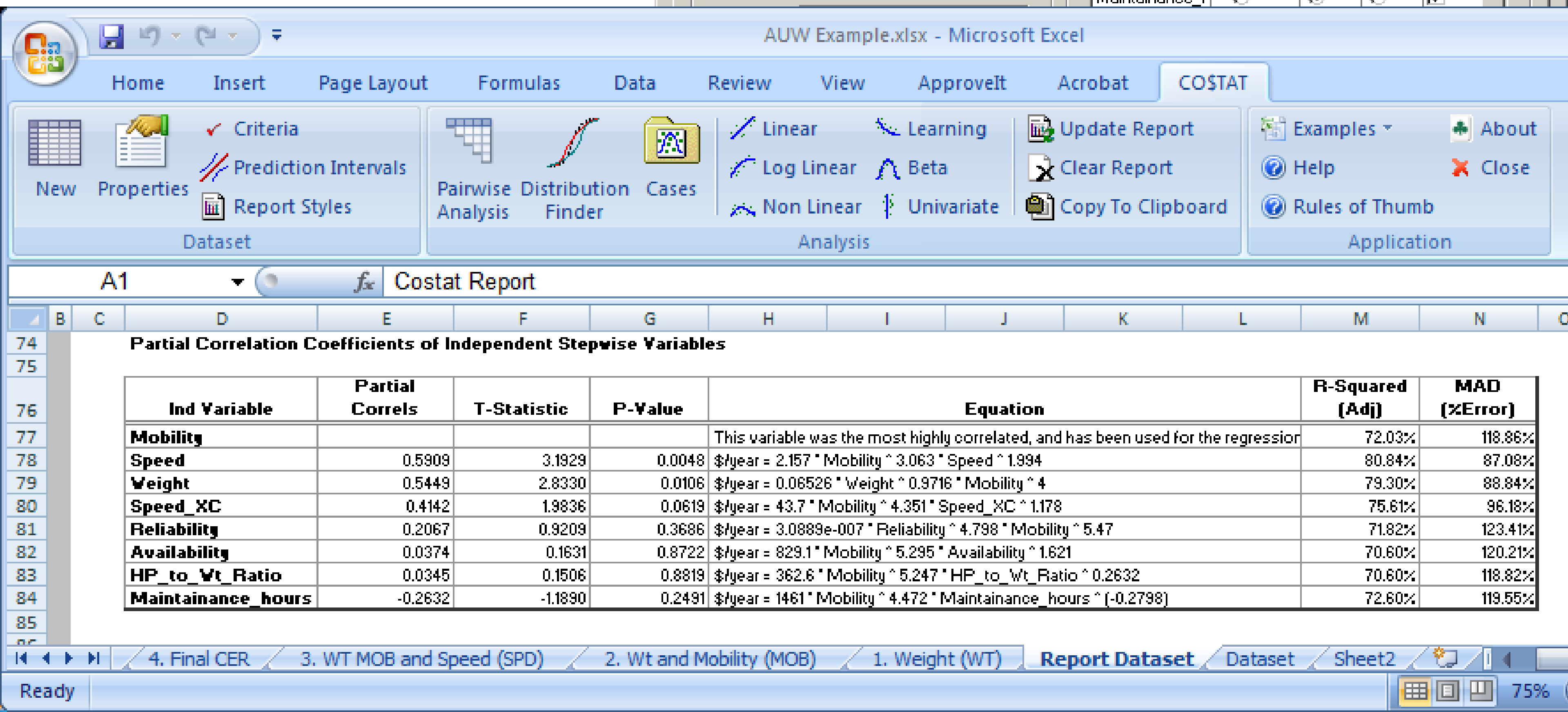
Example of CO$TAT Stepwise Analysis

- Multiple criteria for stopping
  - No further improvement possible
  - Degrees of freedom
  - Multicollinearity
  - Too many variables (variables lose meaning)
  - How will the results be used
- Stopping criteria is a best practices and peer review topic

CO$TAT Stepwise Analysis (Note: Step 0)

Step 0: When no independent variables are selected, CO$TAT will automatically “recommend” the best base variable.



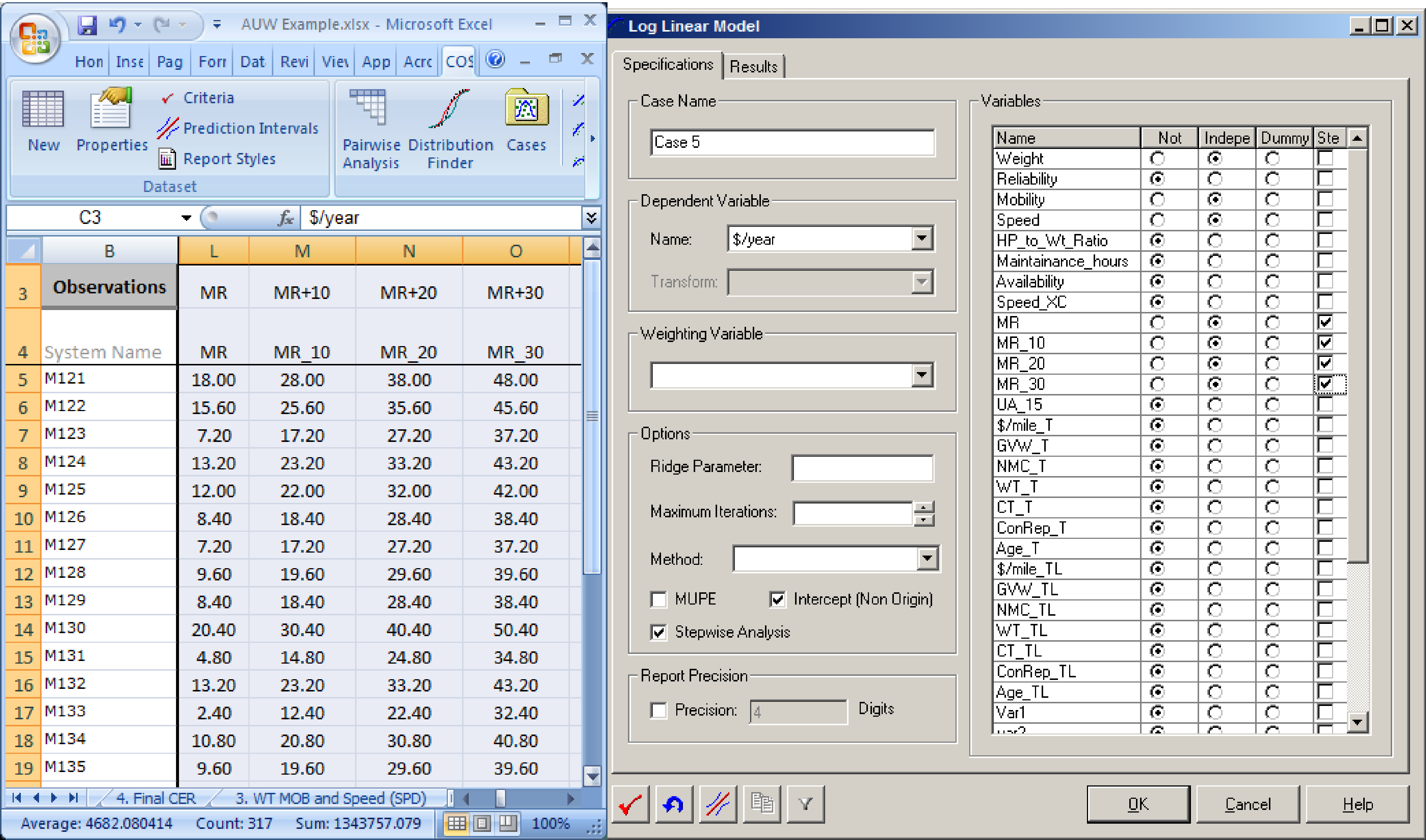


Partial Correlation Coefficients of Independent Stepwise Variables

Ind Variable	Partial Correls	T-Statistic	P-Value	Equation	R-Squared (Adj)	MAD (%Error)
Mobility				This variable was the most highly correlated, and has been used for the regression	72.03%	118.86%
Speed	0.5909	3.1929	0.0048	$/year = 2.157 * Mobility ^ 3.063 * Speed ^ 1.994	80.84%	87.08%
Weight	0.5449	2.8330	0.0106	$/year = 0.06526 * Weight ^ 0.9716 * Mobility ^ 4	79.30%	88.84%
Speed_XC	0.4142	1.9836	0.0619	$/year = 43.7 * Mobility ^ 4.351 * Speed_XC ^ 1.178	75.61%	96.18%
Reliability	0.2067	0.9209	0.3686	$/year = 3.0889e-007 * Reliability ^ 4.798 * Mobility ^ 5.47	71.82%	123.41%
Availability	0.0374	0.1631	0.8722	$/year = 829.1 * Mobility ^ 5.295 * Availability ^ 1.621	70.60%	120.21%
HP_to_Wt_Ratio	0.0345	0.1506	0.8819	$/year = 362.6 * Mobility ^ 5.247 * HP_to_Wt_Ratio ^ 0.2632	70.60%	118.82%
Maintainance_hours	-0.2632	-1.1890	0.2491	$/year = 1461 * Mobility ^ 4.472 * Maintainance_hours ^ (-0.2798)	72.60%	119.55%

CO$TAT Stepwise Analysis (Transforms)

Ability to compare variable transforms quickly



CO$TAT Stepwise Analysis (Transforms)

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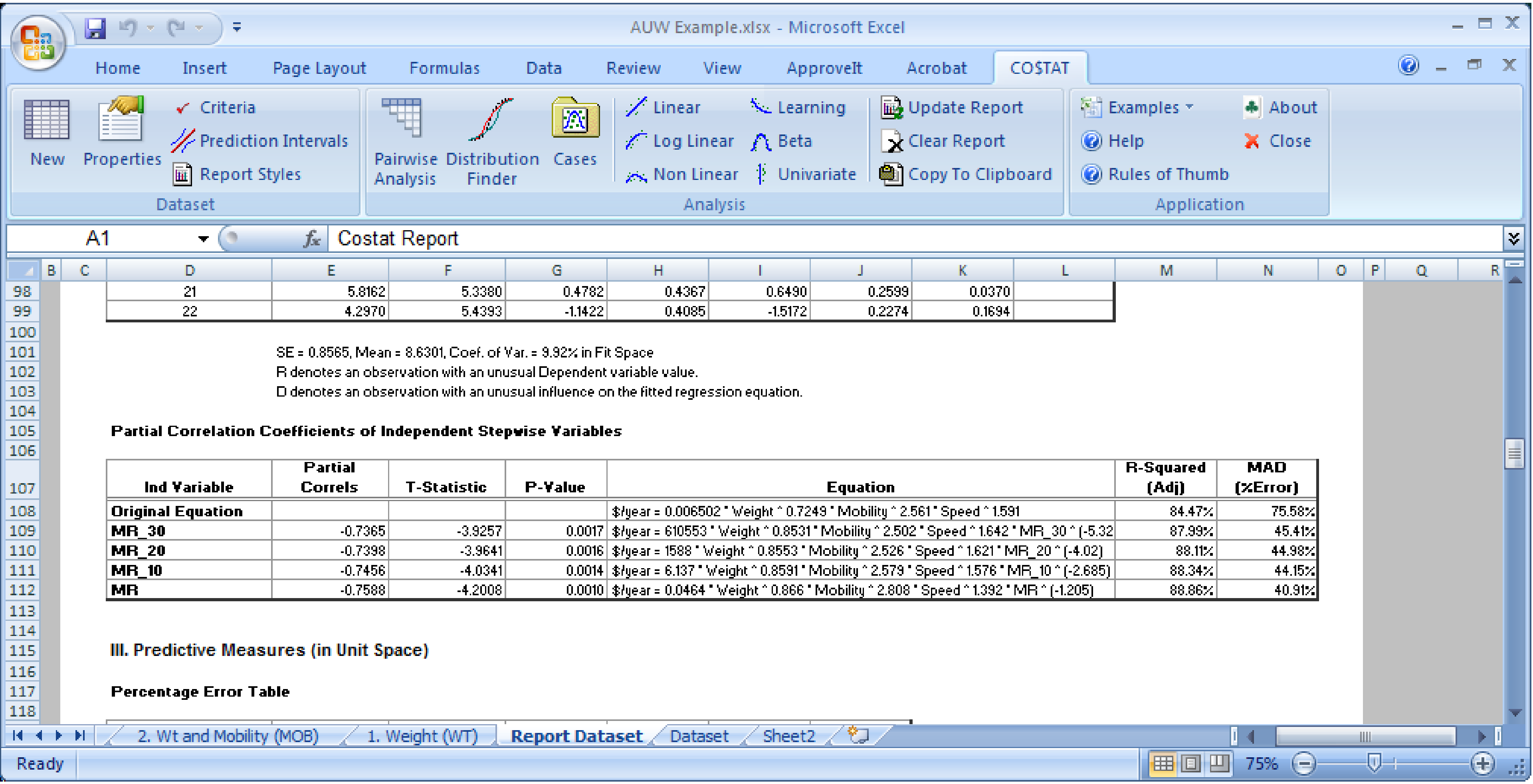


21	5.8162	5.3380	0.4782	0.4367	0.6490	0.2599	0.0370
22	4.2970	5.4393	-1.1422	0.4085	-1.5172	0.2274	0.1694

SE = 0.8565, Mean = 8.6301, Coef. of Var. = 9.92% in Fit Space
R denotes an observation with an unusual Dependent variable value.
D denotes an observation with an unusual influence on the fitted regression equation.

Partial Correlation Coefficients of Independent Stepwise Variables

Ind Variable	Partial Correls	T-Statistic	P-Value	Equation	R-Squared (Adj)	MAD (%Error)
Original Equation				$/year = 0.006502 * Weight ^ 0.7249 * Mobility ^ 2.561 * Speed ^ 1.591	84.47%	75.58%
MR_30	-0.7365	-3.9257	0.0017	$/year = 610553 * Weight ^ 0.8531 * Mobility ^ 2.502 * Speed ^ 1.642 * MR_30 ^ (-5.32	87.99%	45.41%
MR_20	-0.7398	-3.9641	0.0016	$/year = 1588 * Weight ^ 0.8553 * Mobility ^ 2.526 * Speed ^ 1.621 * MR_20 ^ (-4.02)	88.11%	44.98%
MR_10	-0.7456	-4.0341	0.0014	$/year = 6.137 * Weight ^ 0.8591 * Mobility ^ 2.579 * Speed ^ 1.576 * MR_10 ^ (-2.685)	88.34%	44.15%
MR	-0.7588	-4.2008	0.0010	$/year = 0.0464 * Weight ^ 0.866 * Mobility ^ 2.808 * Speed ^ 1.392 * MR ^ (-1.205)	88.86%	40.91%

III. Predictive Measures (in Unit Space)

Percentage Error Table

Advanced Topics

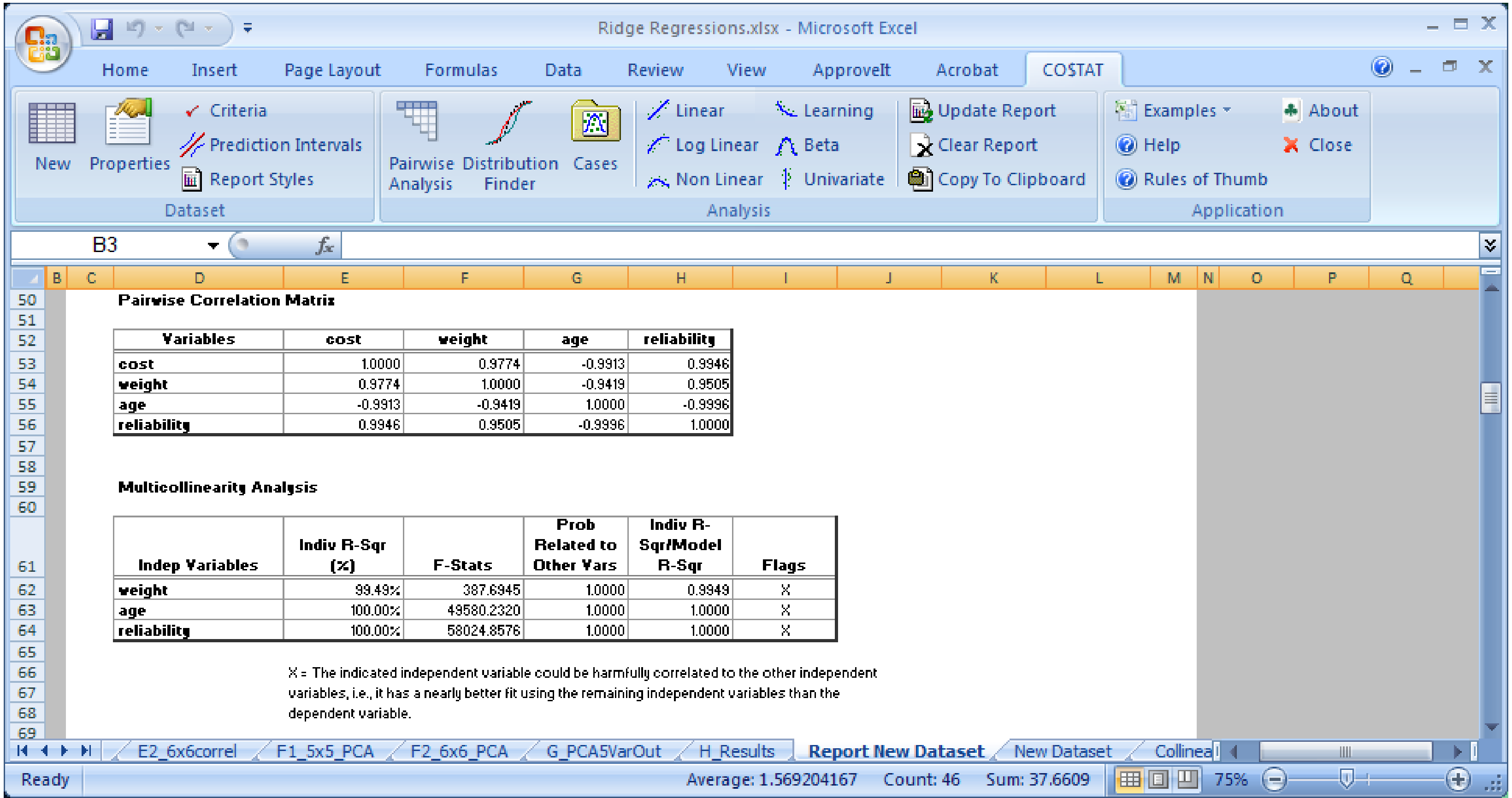
Advanced Topics

Developing best practices for advanced topics:

- Handling Multicollinearity
- Missing Data
- Other advanced topics

Handling Multicollinearity

- True multicollinearity can be avoided
- Residual multicollinearity can be explained
- Should be investigated even when there are no CO$TAT flags



Pairwise Correlation Matrix

Variables	cost	weight	age	reliability
cost	1.0000	0.9774	-0.9913	0.9946
weight	0.9774	1.0000	-0.9419	0.9505
age	-0.9913	-0.9419	1.0000	-0.9996
reliability	0.9946	0.9505	-0.9996	1.0000

Multicollinearity Analysis

Indep Variables	Indiv R-Sqr (%)	F-Stats	Prob Related to Other Vars	Indiv R-Sqr/Model R-Sqr	Flags
weight	99.49%	387.6945	1.0000	0.9949	X
age	100.00%	49580.2320	1.0000	1.0000	X
reliability	100.00%	58024.8576	1.0000	1.0000	X

X = The indicated independent variable could be harmfully correlated to the other independent variables, i.e., it has a nearly better fit using the remaining independent variables than the dependent variable.

Handling Multicollinearity

Consider:

- ACEIT recommendations
- ACEIT Flags
- Defense Acquisition University (DAU) guidance?
- What level of Multicollinearity is OK?
- Is the estimated system in the range of the data?
- Should the coefficients in the CER stand alone?
- What are the predicted values?

Handling Multicollinearity (Lessons Learned)

Techniques:

- Get more data points
- Use fewer variables
- Get better explanatory variables
- Consider meaningful transforms and combinations of the variables
- Ridge Regression
- Principal Component Analysis

Handling Multicollinearity (Explain away)

In this example, we can make a case that the exponent of GVW is meaningful.

Cases

Linear | Log Linear | Non Linear | Learning | Beta | Univariate | Distribution Finder | Analysis Summary

Calc	Type	Name	Status	Equation	DF	F-Prob	T-Prob Intercept	T-Prob b1	T-Prob b2	T-Prob b3	T-Prob b4	R^2 Adj(%)	SE (fit space)
☐	Log Linear	Case 125	Calculated	$/mile = 1.0376e-005 * var1P ^ 3.82	15	1.0000	0.9668	1.0000	0.9475	0.9996	1.0000	93.5826	0.5506
☐	Log Linear	Case 71	Calculated	$/mile = 0.0001523 * GVW ^ 0.5604	12	1.0000	0.9888	0.9296	0.9912	0.9999		90.6174	0.5576
☐	Log Linear	Case 63	Calculated	$/mile = 1.7483e-005 * GVW ^ 0.76	17	1.0000	0.9999	0.9976	0.9966	1.0000		92.1461	0.5759
☐	Log Linear	Case 64	Calculated	$/mile = 1.7483e-005 * GVW ^ 0.76	17	1.0000	0.9999	0.9976	0.9966	1.0000		92.1461	0.5759
☐	Log Linear	Case 61	Calculated	$/mile = 1.7149e-006 * GVW ^ 0.965	17	1.0000	1.0000	0.9997	0.9930	1.0000		92.9730	0.5768
☐	Log Linear	Case 94	Calculated	$/mile = 1.3445e-005 * GVW ^ 0.647	17	1.0000	1.0000	0.9928	0.9814	0.9319	0.9998	92.8923	0.5795
☐	Log Linear	Case 103	Calculated	$/mile = 6.8967e-007 * GVW ^ 0.65	17	1.0000	1.0000	0.9930	0.9794	0.9229	0.9999	92.8147	0.5826
☐	Log Linear	Case 77	Calculated	$/mile = 4.5910e-005 * GVW ^ 0.61	10	1.0000	0.9920	0.9270	0.6047	0.9636	0.9994	89.1117	0.5831
☐	Log Linear	Case 102	Calculated	$/mile = 3.6206e-008 * GVW ^ 0.657	17	1.0000	1.0000	0.9932	0.9782	0.9216	0.9999	92.7728	0.5843
☐	Log Linear	Case 35	Calculated	Cost_mile = 1.9510e-007 * GVW ^ 1.	13	1.0000	0.9996	1.0000				72.2508	0.5873
☐	Log Linear	Case 39	Calculated	Cost_mile = 1.9510e-007 * GVW ^ 1.	13	1.0000	0.9996	1.0000				72.2508	0.5873
☐	Log Linear	Case 46	Calculated	Cost_mile = 1.9510e-007 * GVW ^ 1.	13	1.0000	0.9996	1.0000				72.2508	0.5873
☐	Log Linear	Case 105	Calculated	$/mile = 1.9510e-007 * GVW ^ 1.81	13	1.0000	0.9996	1.0000				72.2508	0.5873
☐	Log Linear	Case 106	Calculated	$/mile = 1.9510e-007 * GVW ^ 1.81	13	1.0000	0.9996	1.0000				72.2508	0.5873
☐	Log Linear	Case 122	Calculated	$/mile = 6.8457e-008 * var1P ^ 3.88	17	1.0000	1.0000	1.0000	0.9997	1.0000	0.9817	92.5298	0.5941
☐	Log Linear	Case 59	Calculated	$/mile = 2.5306e-006 * GVW ^ 0.924	16	1.0000	1.0000	0.9993	0.8538	0.9115	0.9988	92.4363	0.5978
☐	Log Linear	Case 51	Calculated	Cost_mile = 1.8585e-007 * GVW ^ 2.	12	0.9997	0.9994	0.9457	0.3597			70.5033	0.6055
☐	Log Linear	Case 92	Calculated	$/mile = 2.7054e-005 * CW ^ 0.7995	16	1.0000	1.0000	0.9977	0.8695	0.7509	0.9611	92.0897	0.6113
☐	Log Linear	Case 24	Calculated	Cost_mile = 9.8518e-008 * GVW ^ 1.	12	0.9998	0.9995	0.9999	0.5230			71.3532	0.6173
☐	Log Linear	Case 87	Calculated	$/mile = 0.0001176 * GVW ^ 0.8602	17	1.0000	0.9990	0.9985	0.9994	0.9997	0.9993	91.9288	0.6175
☐	Log Linear	Case 127	Calculated	$/mile = 0.0002267 * GVW ^ 0.811 *	16	1.0000	0.9969	0.9966	0.9989	0.9965	0.6051	91.8152	0.6218
☐	Log Linear	Case 128	Calculated	$/mile = 4.2187e-005 * GVW ^ 0.811	16	1.0000	0.9995	0.9966	0.9989	0.6051	0.9992	91.8152	0.6218
☐	Log Linear	Case 89	Calculated	$/mile = 1.9720e-005 * GVW ^ 0.87	17	1.0000	0.9999	0.9984	0.8805	0.9971	0.9988	91.4183	0.6367
☐	Log Linear	Case 99	Calculated	$/mile = 1.9720e-005 * GVW ^ 0.87	17	1.0000	0.9999	0.9984	0.8805	0.9971	0.9988	91.4183	0.6367
☐	Log Linear	Case 100	Calculated	$/mile = 1.9720e-005 * GVW ^ 0.87	17	1.0000	0.9999	0.9984	0.8805	0.9971	0.9988	91.4183	0.6367
☐	Log Linear	Case 101	Calculated	$/mile = 1.9720e-005 * GVW ^ 0.87	17	1.0000	0.9999	0.9984	0.8805	0.9971	0.9988	91.4183	0.6367
☐	Log Linear	Case 22	Calculated	Cost_mile = 2.1636e-007 * Annual_F	11	0.9996	0.9987	0.1876	0.9991	0.0804		67.3290	0.6379
☐	Log Linear	Case 9	Calculated	Cost_mile = 1.5363e-009 * GVW ^ 3.	4	0.8838	0.9412	0.8981	0.4501	0.5023		54.2086	0.6479

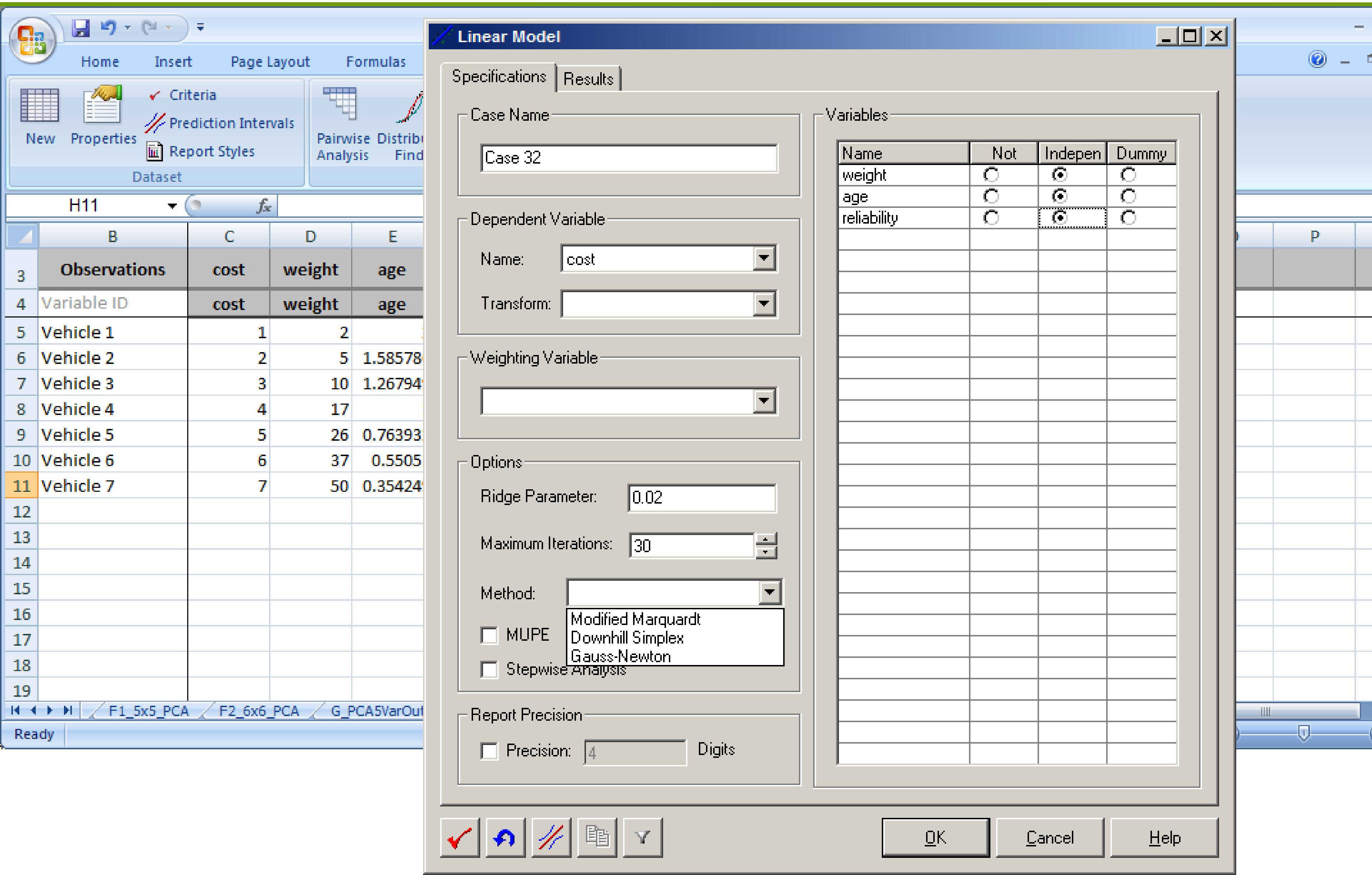
Criteria: None

OK | Cancel | Apply | Help

Handling Multicollinearity (Ridge Regression)

- Already in CO$TAT
- Applies a penalty to sum of squares based on the sum of squares of the coefficients
- Can be used to reduce the effects of multicollinearity, such as counterintuitive coefficients

Handling Multicollinearity (Ridge Regression)

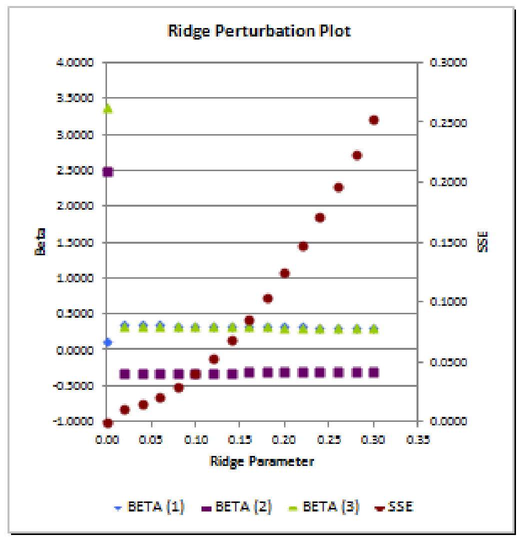


Handling Multicollinearity (Ridge Regression)

Ridge Trace Table

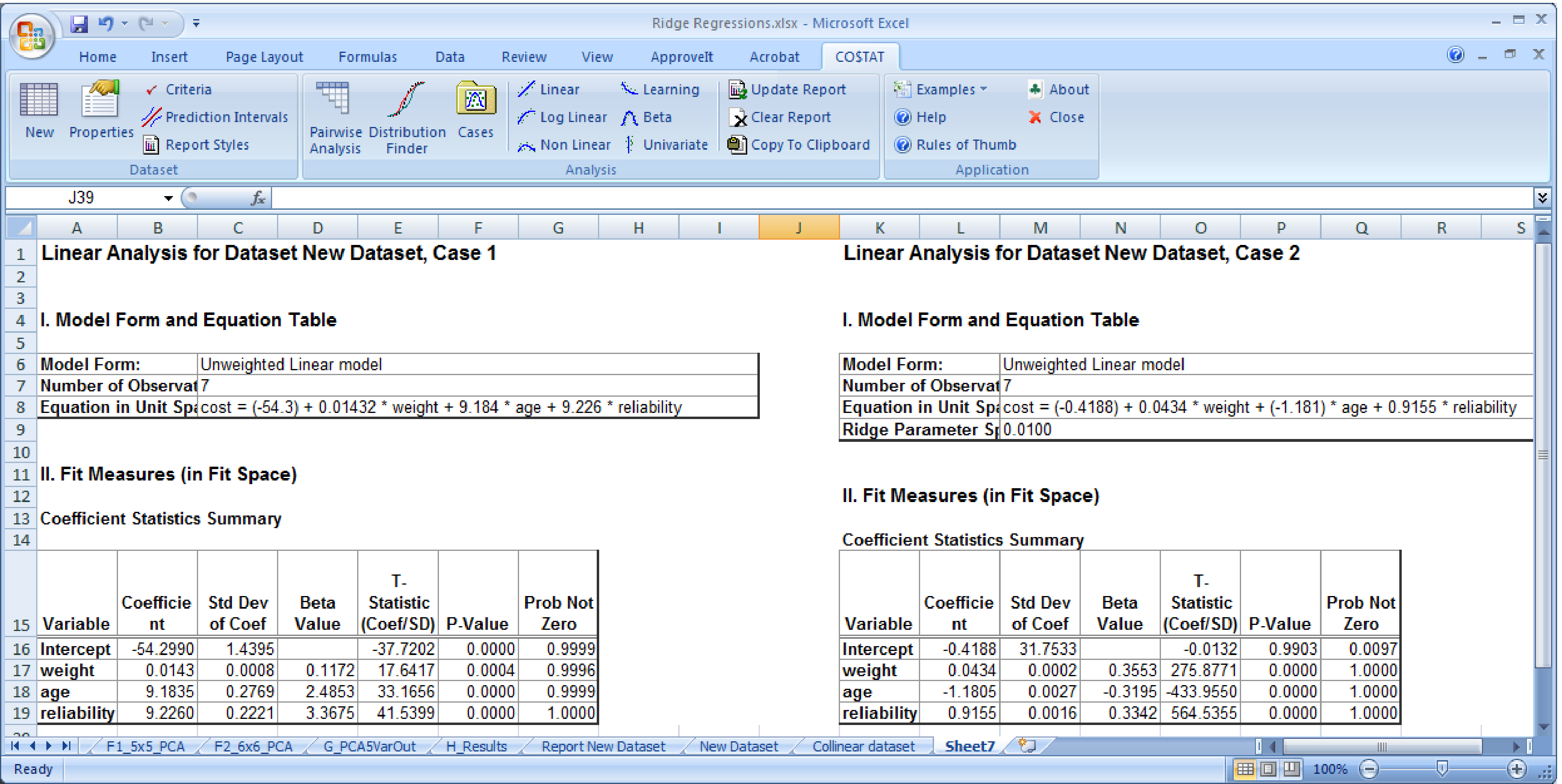
Ridge Parameters	BETA (1)	BETA (2)	BETA (3)	SSE
0.00	0.1172	2.4853	3.3675	0.0000
0.02	0.3518	-0.3224	0.3313	0.0103
0.04	0.3459	-0.3235	0.3293	0.0143
0.06	0.3411	-0.3231	0.3277	0.0207
0.08	0.3370	-0.3222	0.3261	0.0293
0.10	0.3334	-0.3210	0.3244	0.0402
0.12	0.3301	-0.3196	0.3226	0.0532
0.14	0.3270	-0.3180	0.3209	0.0682
0.16	0.3242	-0.3164	0.3191	0.0851
0.18	0.3215	-0.3148	0.3172	0.1039
0.20	0.3189	-0.3131	0.3154	0.1245
0.22	0.3165	-0.3113	0.3136	0.1469
0.24	0.3141	-0.3096	0.3117	0.1708
0.26	0.3118	-0.3079	0.3099	0.1964
0.28	0.3095	-0.3061	0.3081	0.2235
0.30	0.3074	-0.3044	0.3063	0.2521

VII. Charts



Handling Multicollinearity (Ridge Regression)

This example shows the somewhat dramatic effects on the regression with the application of a Ridge Parameter (Case 2). Notice the coefficients in the equation.



Linear Analysis for Dataset New Dataset, Case 1

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observat	7
Equation in Unit Spa	cost = (-54.3) + 0.01432 * weight + 9.184 * age + 9.226 * reliability

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	-54.2990	1.4395		-37.7202	0.0000	0.9999
weight	0.0143	0.0008	0.1172	17.6417	0.0004	0.9996
age	9.1835	0.2769	2.4853	33.1656	0.0000	0.9999
reliability	9.2260	0.2221	3.3675	41.5399	0.0000	1.0000

Linear Analysis for Dataset New Dataset, Case 2

I. Model Form and Equation Table

Model Form:	Unweighted Linear model
Number of Observat	7
Equation in Unit Spa	cost = (-0.4188) + 0.0434 * weight + (-1.181) * age + 0.9155 * reliability
Ridge Parameter S	0.0100

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	P-Value	Prob Not Zero
Intercept	-0.4188	31.7533		-0.0132	0.9903	0.0097
weight	0.0434	0.0002	0.3553	275.8771	0.0000	1.0000
age	-1.1805	0.0027	-0.3195	-433.9550	0.0000	1.0000
reliability	0.9155	0.0016	0.3342	564.5355	0.0000	1.0000

Handling Multicollinearity – Principal Component Analysis (PCA)

- Transforms the variables into new orthogonal variables
- Computationally tricky (requires calculating eigenvalues of the pairwise correlation matrix)
- New variables lose meaning, but the cumulative loading of each variable can be used to recommend suitable variable bases

Handling Multicollinearity – Principal Component Analysis (PCA)

Transformed variables have no pairwise correlation.

Pairwise Correlation Matrix

Variables	$/mile	Var1	var2	var3	var4	var5
$/mile	1.0000	0.7374	-0.0288	0.5393	0.1262	0.1382
Var1	0.7374	1.0000	0.0000	0.0000	0.0000	-0.0002
var2	-0.0288	0.0000	1.0000	0.0000	0.0000	0.0000
var3	0.5393	0.0000	0.0000	1.0000	0.0000	0.0000
var4	0.1262	0.0000	0.0000	0.0000	1.0000	0.0000
var5	0.1382	-0.0002	0.0000	0.0000	0.0000	1.0000

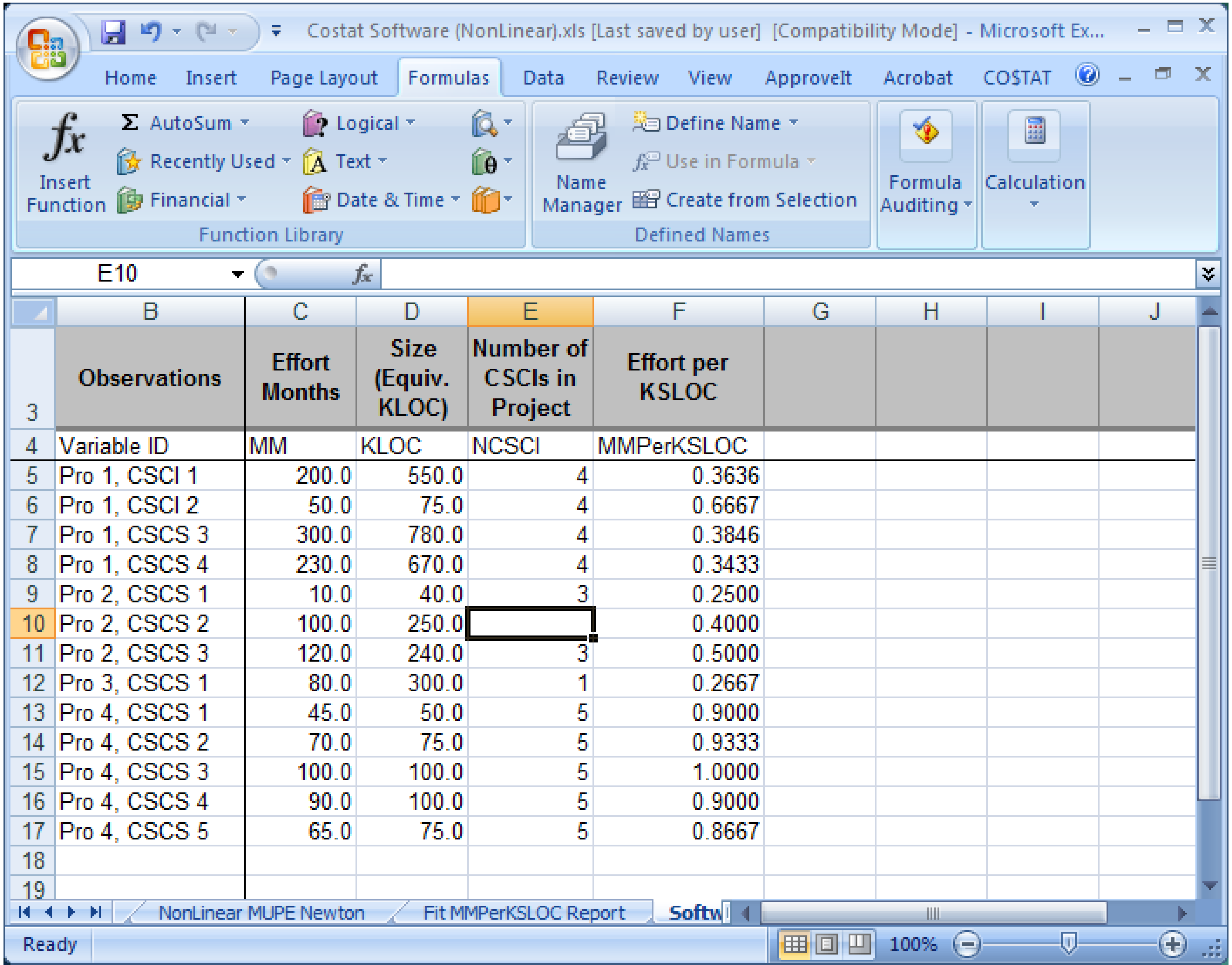
Multicollinearity Analysis

Indep Variables	Indiv R-Sqr (%)	F-Stats	Prob Related to Other Vars	Indiv R-Sqr/Model R-Sqr	Flags
Var1	0.00%	0.0000	0.0000	0.0000	
var2	0.00%	0.0000	0.0000	0.0000	
var3	0.00%	0.0000	0.0000	0.0000	
var4	0.00%	0.0000	0.0000	0.0000	
var5	0.00%	0.0000	0.0000	0.0000	

Outlier Analysis Summary

Incomplete Data

An example from CO$TAT examples.



	B	C	D	E	F	G	H	I	J
3	Observations	Effort Months	Size (Equiv. KLOC)	Number of CSCIs in Project	Effort per KSLOC				
4	Variable ID	MM	KLOC	NCSCI	MMPerKSLOC				
5	Pro 1, CSCI 1	200.0	550.0	4	0.3636				
6	Pro 1, CSCI 2	50.0	75.0	4	0.6667				
7	Pro 1, CSCS 3	300.0	780.0	4	0.3846				
8	Pro 1, CSCS 4	230.0	670.0	4	0.3433				
9	Pro 2, CSCS 1	10.0	40.0	3	0.2500				
10	Pro 2, CSCS 2	100.0	250.0		0.4000				
11	Pro 2, CSCS 3	120.0	240.0	3	0.5000				
12	Pro 3, CSCS 1	80.0	300.0	1	0.2667				
13	Pro 4, CSCS 1	45.0	50.0	5	0.9000				
14	Pro 4, CSCS 2	70.0	75.0	5	0.9333				
15	Pro 4, CSCS 3	100.0	100.0	5	1.0000				
16	Pro 4, CSCS 4	90.0	100.0	5	0.9000				
17	Pro 4, CSCS 5	65.0	75.0	5	0.8667				
18									
19									

Incomplete Data

- Most users believe in throwing out the observation or disregarding the variable
- As is, CO$TAT treats the missing point as a 0
- Other techniques are available
- On a limited basis, the data can still be used acceptably
- Working on techniques and best practices for this case

Other Advanced Topics

- Sensitivity analysis
  - Input data
  - Omission of a data point
  - Communicating sensitivity results
- Pooled Regression and Dummy Variables (tomorrow)
- Using Minimum-Unbiased-Percentage Error (CO$TAT feature)

Developing Best Practices

Developing Best Practices

- Best Practices in CO$TAT is extremely challenging
- Rules are difficult to quantify
- Best practices should consist of a series of “world-class” well-documented illustrations
- Target experienced users
- Data sources and documentation of data sources is critical
- Use peer reviews

Summary

- CO$TAT Stepwise Analysis tool benefits the analyst
  - Time and effectiveness
  - Documentation
  - Defending results
- Consider building best practices for Stepwise Analysis
  - General guidance
  - Advanced topics (expand the analyst toolset)
- Peer review is a critical component of best practices

Questions