Using Unitized Uncertainty Distributions in ACEIT

ACEIT Users Workshop
February 1-2, 2010
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Outline

- Background and Definition
- Simple Example Exercise
- Refined Definition
- Catalog
- Expanded Exercise
- Best Practices
- Summary

Background and Definition

Challenge

- There is an ever-present desire for general-purpose distributions to apply to WBS elements by commodity.
- Ongoing research sponsored by the Air Force Cost Analysis Agency will meet this need.
- Product will consist of a metrics manual of unitized distributions organized by commodity and WBS.

Uncertainty Modeling Algebraically

Given:

Cost Element $_{\text{Point Estimate}}$ = Your Methodology

Its uncertainty can then be expressed:

Cost Element $_{\text{Uncertainty}}$ = *f*(Your Methodology, Distribution Shape, Distribution Parameters)

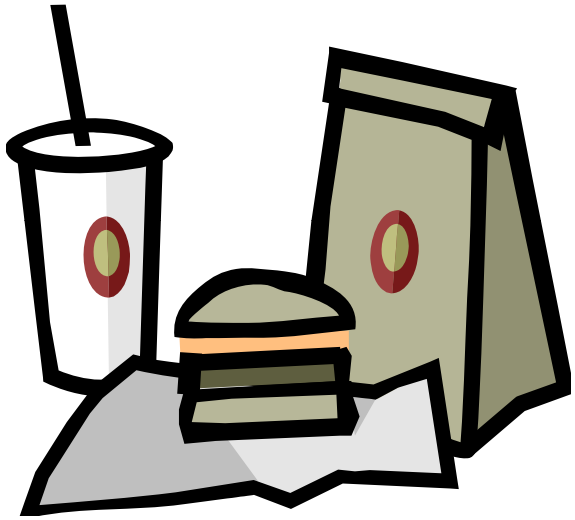
- **Situation-specific**
- **Not Reusable**

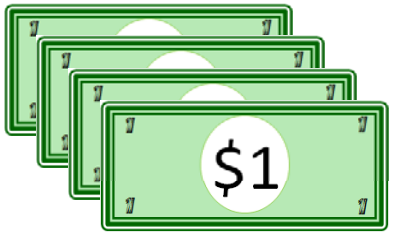
Unitized Distribution: Short Definition

- **A distribution with a center value of one.**

A Simple Example

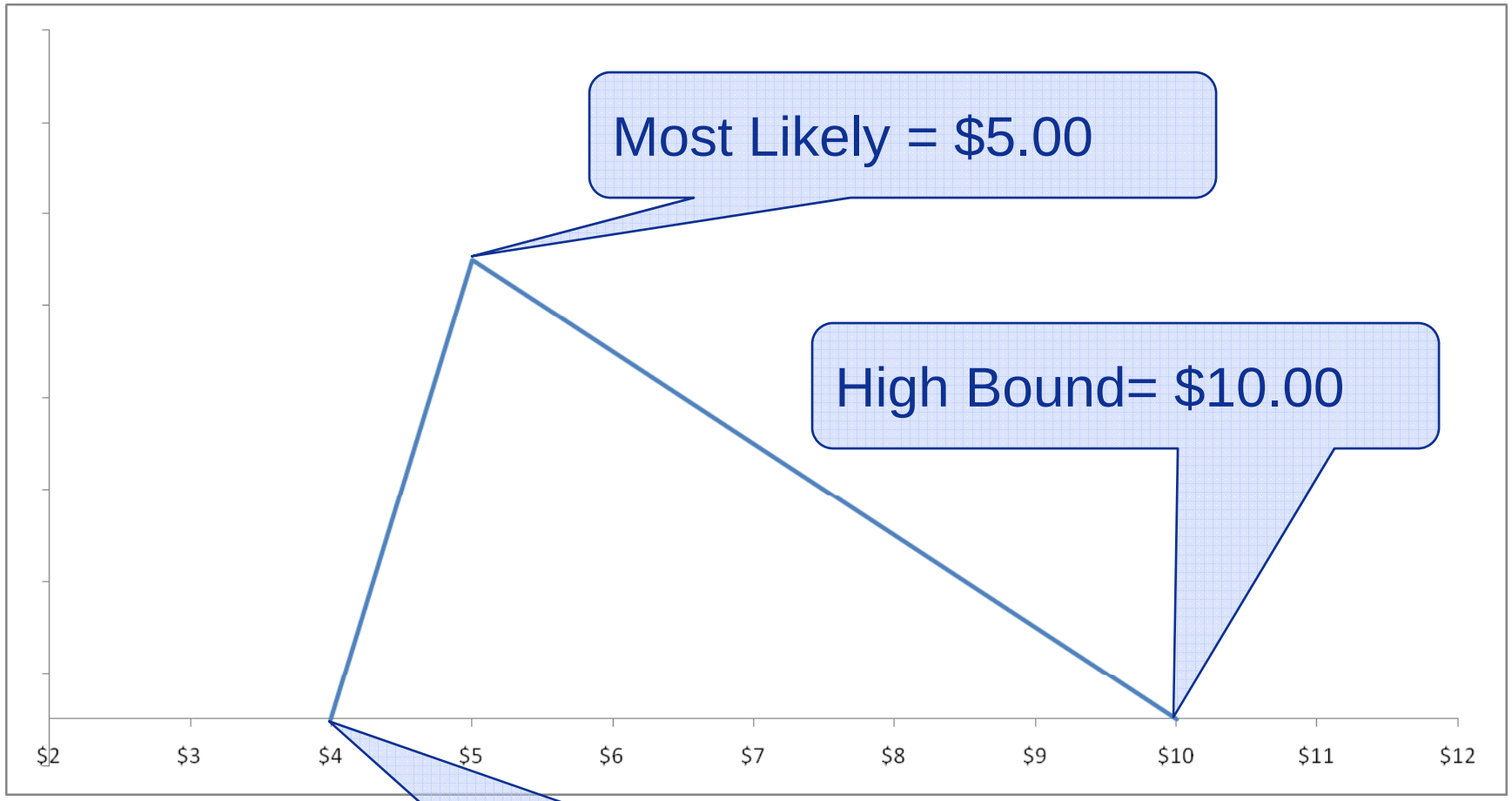
Exercise: Estimate the Cost of Lunch





Range of Outcomes

Conceptual Cost of Lunch



Mean = (min + mode + max)/3
Mean = (4 + 5+ 10)/3 = **6.333**
Std Dev = sqrt((min^2 + $mode^2$ + max^2 – min*mode – min*max – mode*max)/18)
Std Dev = sqrt((4^2 + 5^2 + 10^2 – 4*5 – 4*10 – 5*10)/18) = **1.312**

Algebraic Cost of Lunch

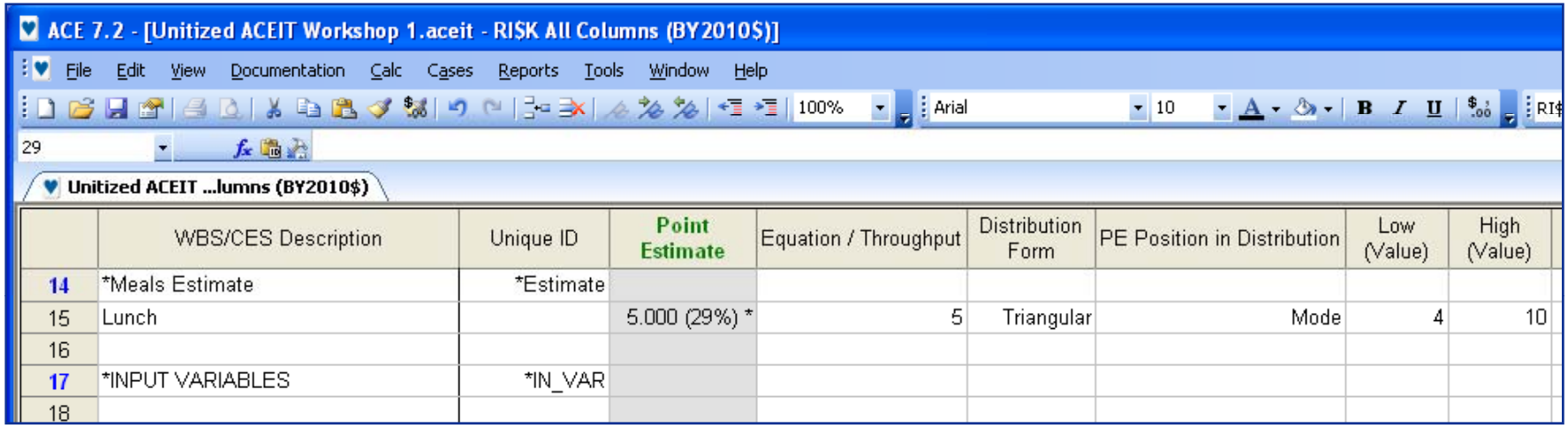
Given:

$$Lunch_{Point\ Estimate} = 5.00$$

Its uncertainty can then be expressed:

$$Lunch_{Uncertainty} = f(\text{Triangular, Low=4.00, ML=5.00, High=10.00})$$

Modeled Cost of Lunch

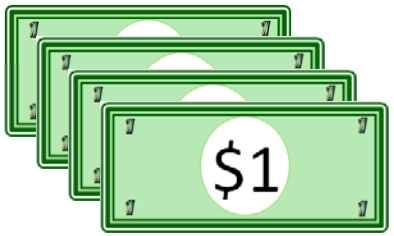


	WBS/CES Description	Unique ID	Point Estimate	Equation / Throughput	Distribution Form	PE Position in Distribution	Low (Value)	High (Value)
14	*Meals Estimate	*Estimate						
15	Lunch		5.000 (29%) *	5	Triangular	Mode	4	10
16								
17	*INPUT VARIABLES	*IN_VAR						
18								

- **All well and good, but:**
  - Situation-specific
  - Not Reusable

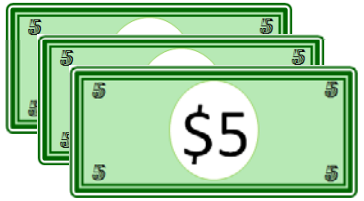
New Exercises

- **Estimate the Cost of Breakfast**



$4.00

- **Estimate the Cost of Dinner**



$15.00

??? Range of Outcomes ???

Algebraic Cost of Breakfast and Dinner

Given:

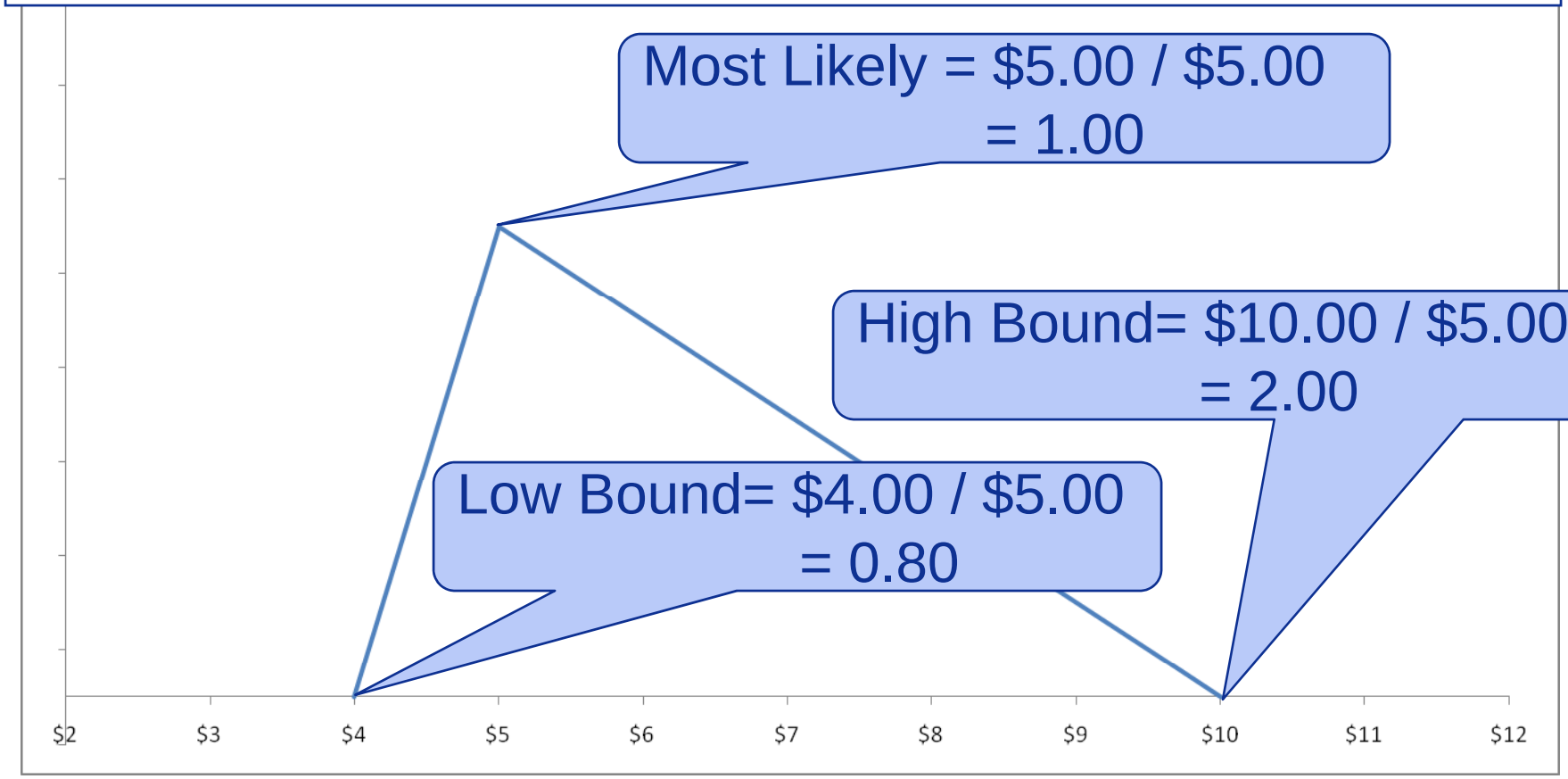
- $\text{Breakfast}_{\text{Point Estimate}} = 4.00$
- $\text{Dinner}_{\text{Point Estimate}} = 15.00$

Insufficient information to express their uncertainty:

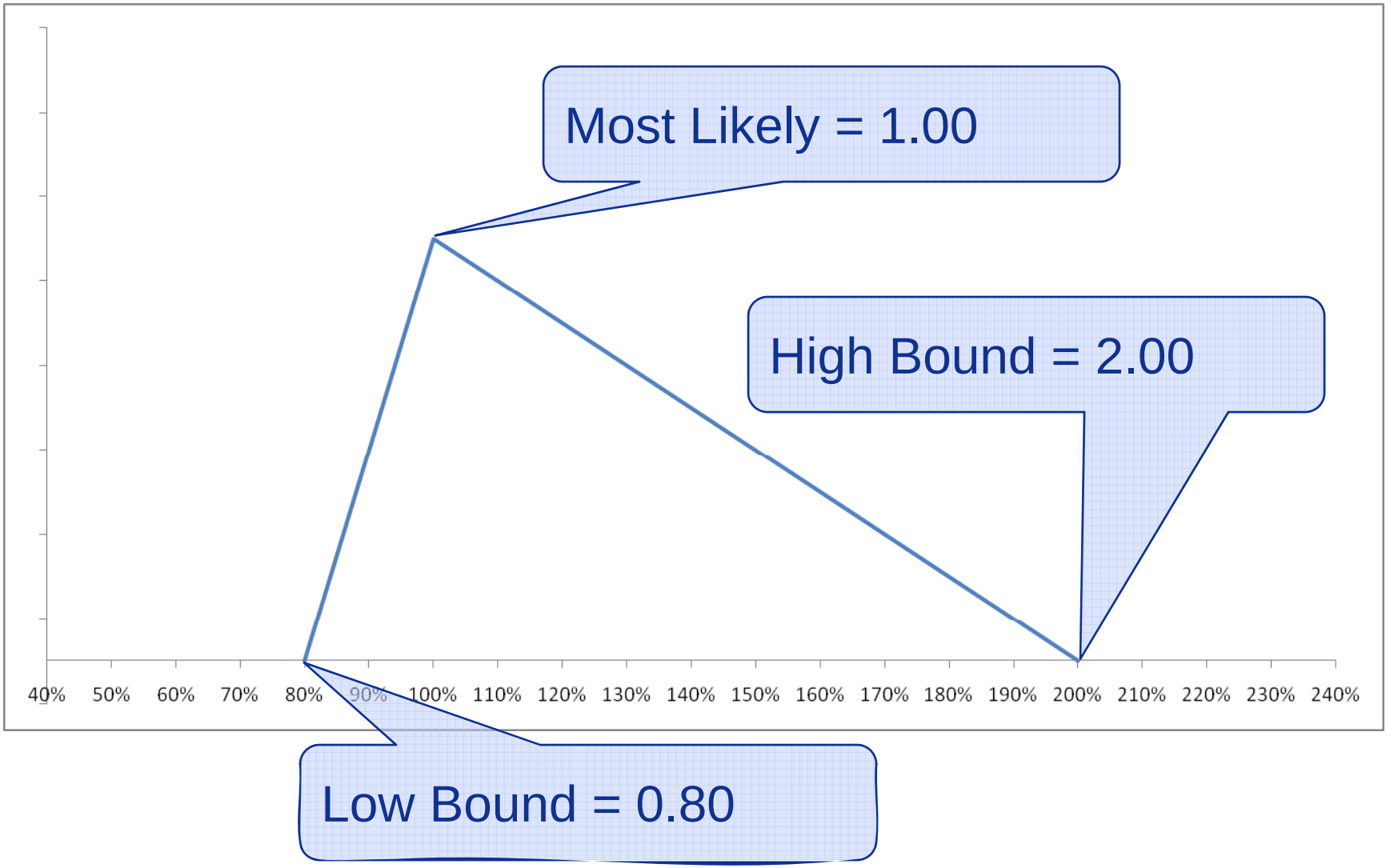
- $\text{Breakfast}_{\text{Uncertainty}} = f(\text{PE}=4, \text{Shape Unknown}, \text{Bounds Unknown})$
- $\text{Dinner}_{\text{Uncertainty}} = f(\text{PE}=15, \text{Shape Unknown}, \text{Bounds Unknown})$

Solution: Borrow Shape and Bounds From a Known Case

- **Cannot use the known case's parameters directly**
- **Can only use their relative values**
- **Use Linear Transform: Divide through by the central value**



Unitized Distribution



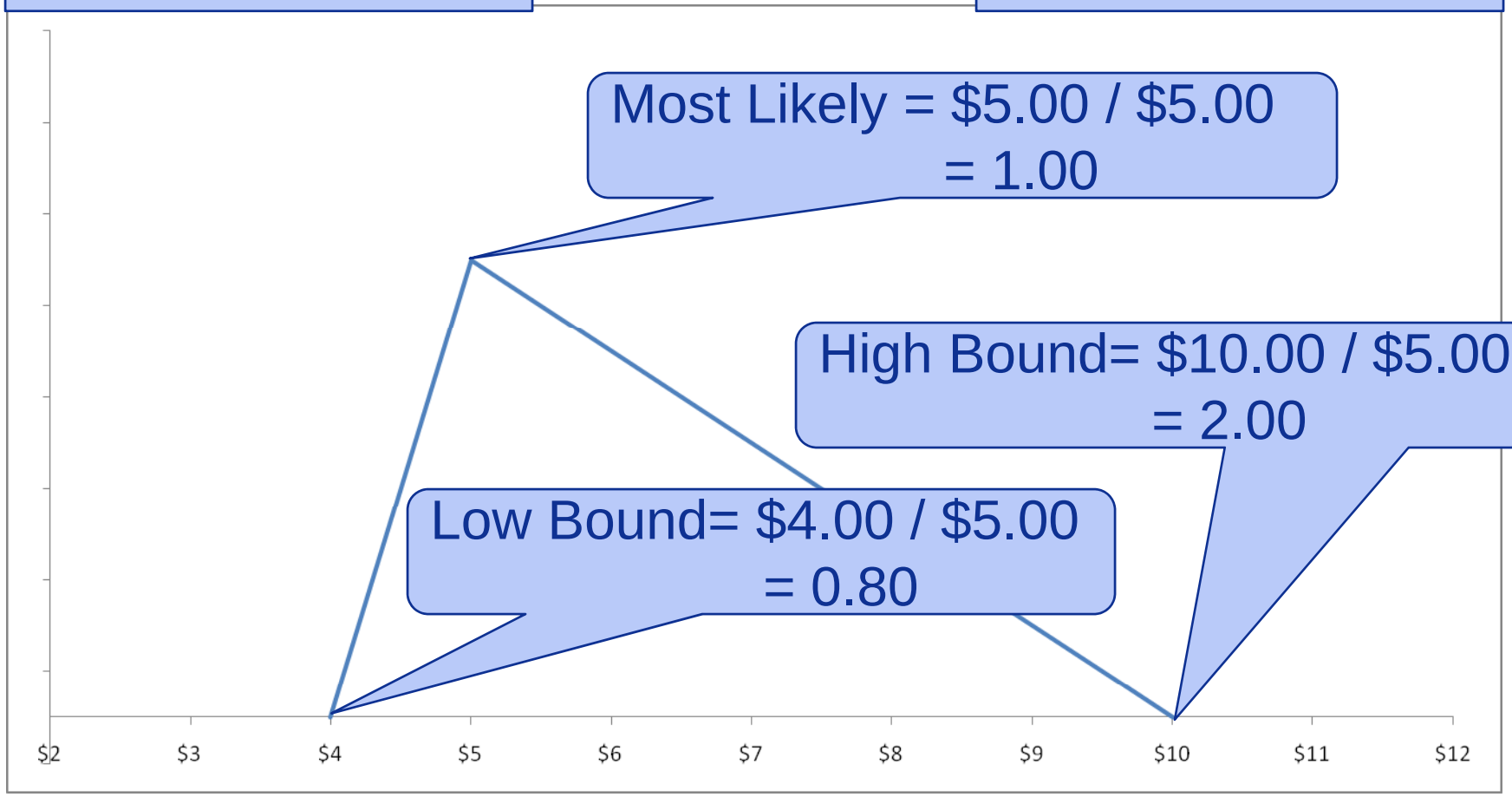
Note Other Characteristics Also Scale Linearly

Mean = (min + mode + max)/3
Mean = (4 + 5+ 10)/3 = **6.333**
Std Dev = sqrt((min^2 + mode^2 + max^2 – min*mode – min*max – mode*max)/18)
Std Dev = sqrt((4^2 + 5^2 + 10^2 – 4*5 – 4*10 – 5*10)/18) = **1.312**

Mean = 6.333 / 5 = **1.267**

Std Dev = 1.312 / 5 = **0.262**

Mean = (min + mode + max)/3
Mean = (0.8 + 1 + 2)/3 = **1.267**
Std Dev = sqrt((min^2 + mode^2 + max^2 – min*mode – min*max – mode*max)/18)
Std Dev = sqrt((0.8^2 + 1^2 + 2^2 – 0.8*1 – 0.8*2 – 1*2)/18) = **0.262**



Algebraic Cost of Breakfast and Dinner

Given:

$\text{Breakfast}_{\text{Point Estimate}} = 4.00$

$\text{Lunch}_{\text{Point Estimate}} = 5.00$

$\text{Dinner}_{\text{Point Estimate}} = 15.00$

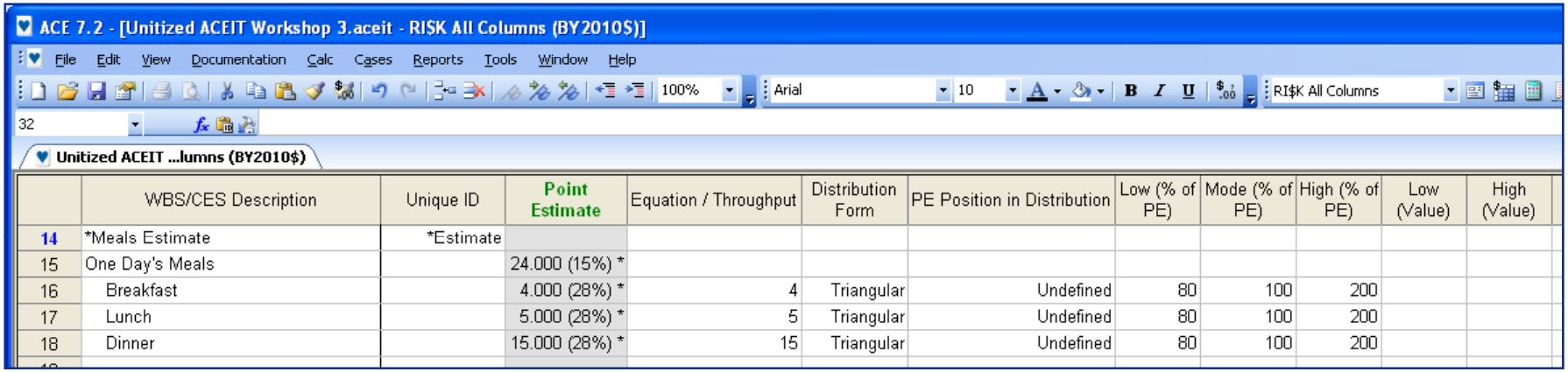
Using Unitized Distributions to express uncertainty:

$\text{Breakfast}_{\text{Uncertainty}} = (PE=4) * \text{Triangular}(0.80, 1.00, 2.00)$

$\text{Lunch}_{\text{Uncertainty}} = (PE=5) * \text{Triangular}(0.80, 1.00, 2.00)$

$\text{Dinner}_{\text{Uncertainty}} = (PE=15) * \text{Triangular}(0.80, 1.00, 2.00)$

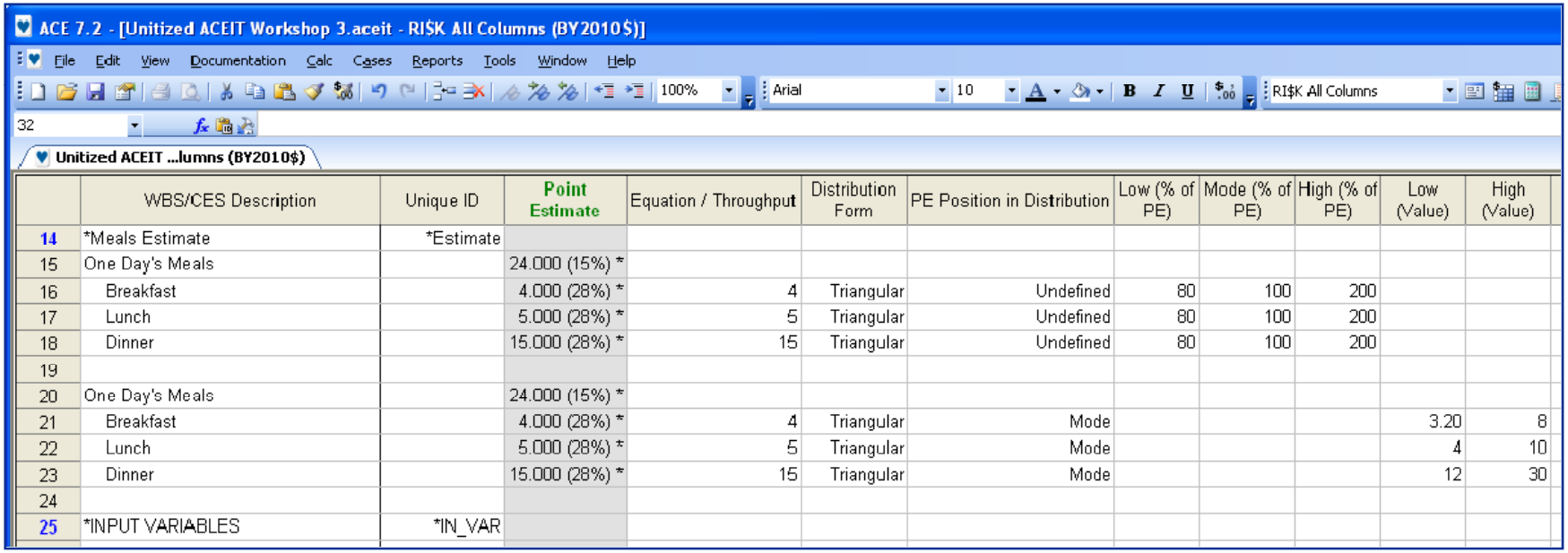
Modeled Cost of Meals



	WBS/CES Description	Unique ID	Point Estimate	Equation / Throughput	Distribution Form	PE Position in Distribution	Low (% of PE)	Mode (% of PE)	High (% of PE)	Low (Value)	High (Value)
14	*Meals Estimate	*Estimate									
15	One Day's Meals		24.000 (15%) *								
16	Breakfast		4.000 (28%) *	4	Triangular	Undefined	80	100	200		
17	Lunch		5.000 (28%) *	5	Triangular	Undefined	80	100	200		
18	Dinner		15.000 (28%) *	15	Triangular	Undefined	80	100	200		

Compare Discrete and Unitized

- **For reference, note unitized approach and discrete approach yields the same result**



ACE 7.2 - [Unitized ACEIT Workshop 3.aceit - RI$K All Columns (BY2010$)]

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Unitized ACEIT ...lumns (BY2010$)

	WBS/CES Description	Unique ID	Point Estimate	Equation / Throughput	Distribution Form	PE Position in Distribution	Low (% of PE)	Mode (% of PE)	High (% of PE)	Low (Value)	High (Value)
14	*Meals Estimate	*Estimate									
15	One Day's Meals		24.000 (15%) *								
16	Breakfast		4.000 (28%) *	4	Triangular	Undefined	80	100	200		
17	Lunch		5.000 (28%) *	5	Triangular	Undefined	80	100	200		
18	Dinner		15.000 (28%) *	15	Triangular	Undefined	80	100	200		
19											
20	One Day's Meals		24.000 (15%) *								
21	Breakfast		4.000 (28%) *	4	Triangular	Mode				3.20	8
22	Lunch		5.000 (28%) *	5	Triangular	Mode				4	10
23	Dinner		15.000 (28%) *	15	Triangular	Mode				12	30
24											
25	*INPUT VARIABLES	*IN_VAR									

Algebraic Using Unitized Distributions

Given:

Cost Element $_{\text{Point Estimate}}$ = Your Methodology

Its uncertainty can then be expressed:

~~Cost Element $_{\text{Uncertainty}}$ = *f*(Your Methodology, Distribution Shape, Distribution Parameters)~~

Cost Element $_{\text{Uncertainty}}$ =

Your Methodology * Unitized Distribution

Refined Definition

Definition

- A unitized distribution has a center value of one.
- A unitized distribution is designed to be modeled as a multiplier of point estimates.
- A unitized distribution is useful when discrete bounds or distribution shape is unknown.

Catalog of Unitized Distributions

Catalog

- **The AFCAA Cost Risk and Uncertainty Analysis Metrics Manual (CRUAMM) will provide guidelines and empirical metrics for developing cost uncertainty analyses**
- **A Catalog of Empirically-Based Unitized Uncertainty Distributions**

	Sample	My Point Estimate is the:		
Dataset	CV	Mean	Median	Mode
WBS # and Stratification Class	nn	Lognormal (Mean, Std Dev)	Lognormal (Mean, Std Dev)	Lognormal (Mean, Std Dev)
WBS # and Stratification Class	nn	Normal (Mean, Std Dev)	Normal (Mean, Std Dev)	Normal (Mean, Std Dev)
WBS # and Stratification Class	nn	Triangular (Low, Mode, High)	Triangular (Low, Mode, High)	Triangular (Low, Mode, High)
WBS # and Stratification Class	nn	Beta (Low, High, Alpha, Beta)	Beta (Low, High, Alpha, Beta)	Beta (Low, High, Alpha, Beta)

Catalog and ACEIT Instructions

Dataset	Sample CV	My Point Estimate is the: Mean	Median	Mode
WBS # and Stratification Class	nn	Normal (Mean, Std Dev)	Normal (Mean, Std Dev)	Normal (Mean, Std Dev)
WBS # and Stratification Class	nn	Lognormal (Mean, Std Dev)	Lognormal (Mean, Std Dev)	Lognormal (Mean, Std Dev)
WBS # and Stratification Class	nn	Triangular (Low, Mode, High)	Triangular (Low, Mode, High)	Triangular (Low, Mode, High)
WBS # and Stratification Class	nn	Beta (Low, High, Alpha, Beta)	Beta (Low, High, Alpha, Beta)	Beta (Low, High, Alpha, Beta)

Distribution Type	PE Position	Priority 1	Priority 2	Priority 3	Priority 4	Priority 5	Priority 6	Priority 7
Normal	Mean/ Median/ Mode	CV	SD	Sp	H	L		
	Low	High						
Log Normal	Mean/ Median/ Mode	ASE	CV	SD	Sp	H	L	
	Low	High						
Triangular (See Note 1)	Mode	L,H	Mode%,H *or* Mode%,L	Sk,H *or* Sk,L	SD,H *or* SD,L	Sp,H *or* Sp,L	Mode%,CV *or* Mode%,SD *or* Mode%,Sp	Sk,CV *or* Sk,SD *or* Sk,Sp
Beta	Mode	CV,alpha, beta	L,H,alpha, beta	L,H	alpha,beta	H,alpha, beta	L,alpha, beta	
Uniform (see Note 2)	Mode	Mode%,CV *or* Mode%,SD *or* Mode%,Sp	Sk,CV *or* Sk,SD *or* Sk,Sp	Mode%,H *or* Sk,H	Mode%,L *or* Sk,L			
	Mean/ Median	CV *or* SD *or* Sp *or* H						
	Undefined	CV, H *or* SD, H *or* Sp, H						
	Low	H						
Weibull (See Note 3)	Mode	Shape, Scale	L, H	Sp	Mode % *or* L,Sk			

Legend:
L = Low (Value) or Low (% of PE)
H = High Value or High (% of PE)
Note that you should also enter Low Percentile and High Percentile when entering Low and/or High values.
Sp = Spread
Sk = Skew
ASE = Adjusted SE
CV = Coefficient of Variation
SD = Standard Deviation
Mode = Most likely value
Mode% = Confidence probability of the mode

Note 1:
For the **Triangular** distribution, enter the confidence level of the mode in the Mode % column. The confidence must be between 0.0 and 1.0. Enter the PE variation with fixed range in the Spread field.

Note 2:
For the **Uniform** distribution, enter the confidence level of the input cost in the Mode% column. The confidence must be between 0.0 and 1.0. Even more specifications for Uniform are allowed. See help topic for Uniform for the complete list.

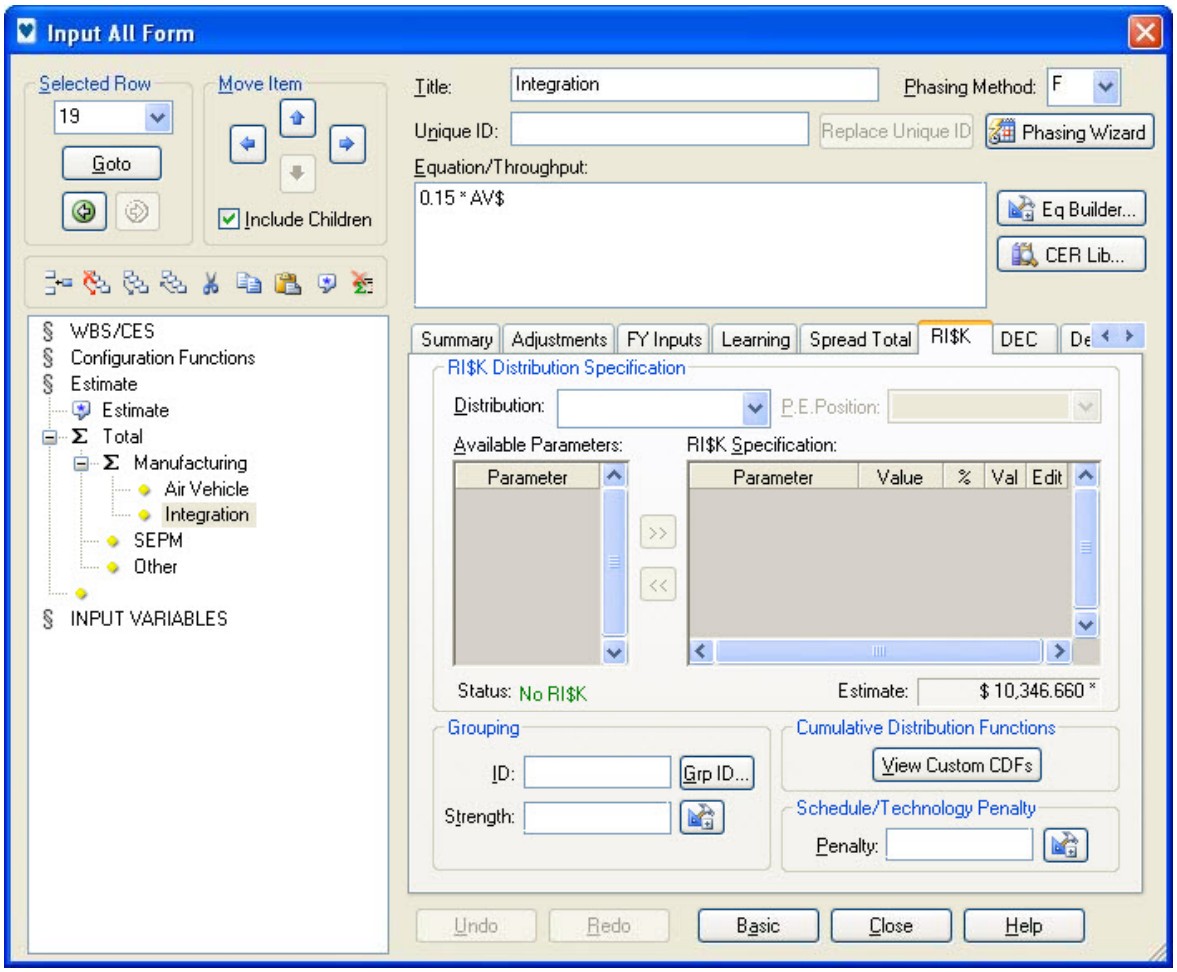
Note 3:
For **Weibull** distribution, partial inputs have different meanings depending on the fields entered. For example, if spread is given alone, it must be a preset selection (i.e., L, M or H). Values for Scale (b) must be between 0.0001 and 30000.0. Range for Shape (alpha) is 1.0 to 300.0. For the Priority 4 case, the Low Percentile is always 0.0%, and the Skew is the confidence of the mode. This value must be between 0.0 and 0.5. See Weibull Distribution for more information.

Source: ACEIT Help

Use the Input All Form's RI$K Tab to Enter Distribution Information

- **Steps in defining a distribution:**
  - **Step 1:** Enter the method or throughput for the row.
  - **Step 2:** Select a distribution type – What is the shape of the uncertainty you want to model?
  - **Step 3:** Enter the Point Estimate (PE) Position – What does the point estimate represent (i.e. the mode, low, high, median, unknown)?
  - **Step 4:** Indentify the remaining shape of the distribution – This will vary depending on the PE position and the bound information you have available to you. **When the distribution is fully specified the status will say Complete.**

Advanced mode of the Input All From RI$K tab

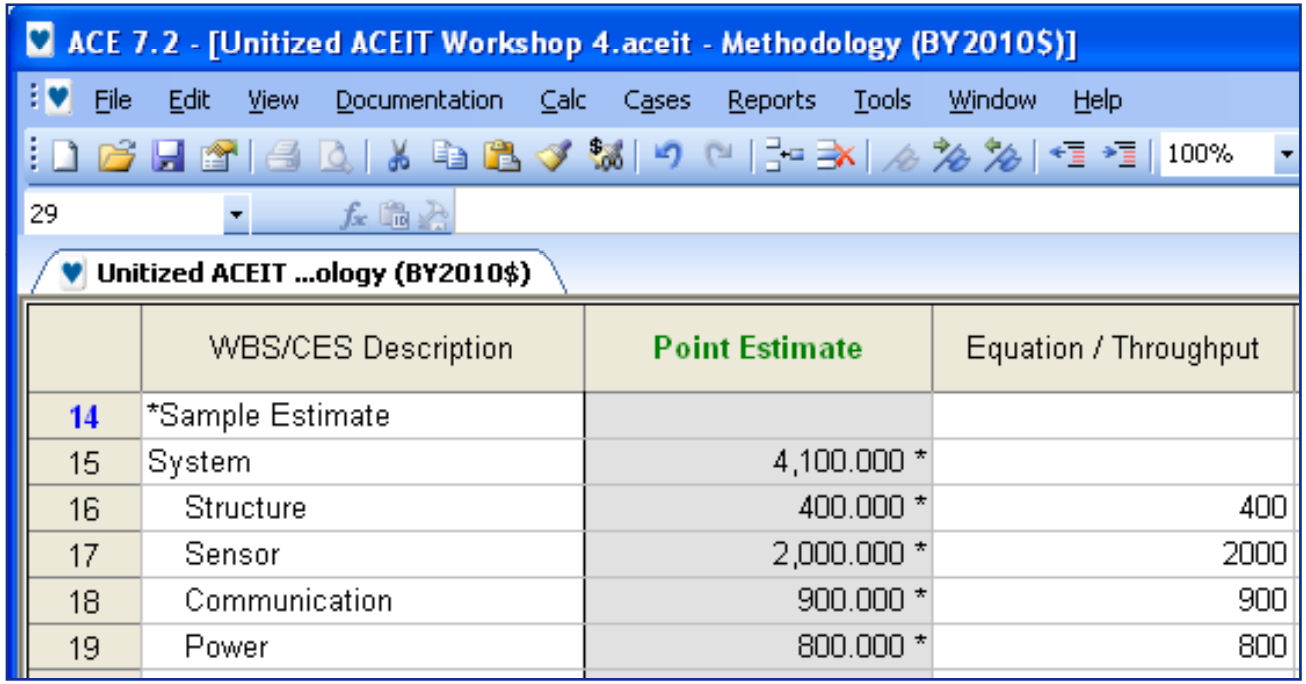


Source: RI$K Training 12 Apr 2010

An Expanded Example

Example

- **Begin with a point estimate**



	WBS/CES Description	Point Estimate	Equation / Throughput
14	*Sample Estimate		
15	System	4,100.000 *	
16	Structure	400.000 *	400
17	Sensor	2,000.000 *	2000
18	Communication	900.000 *	900
19	Power	800.000 *	800

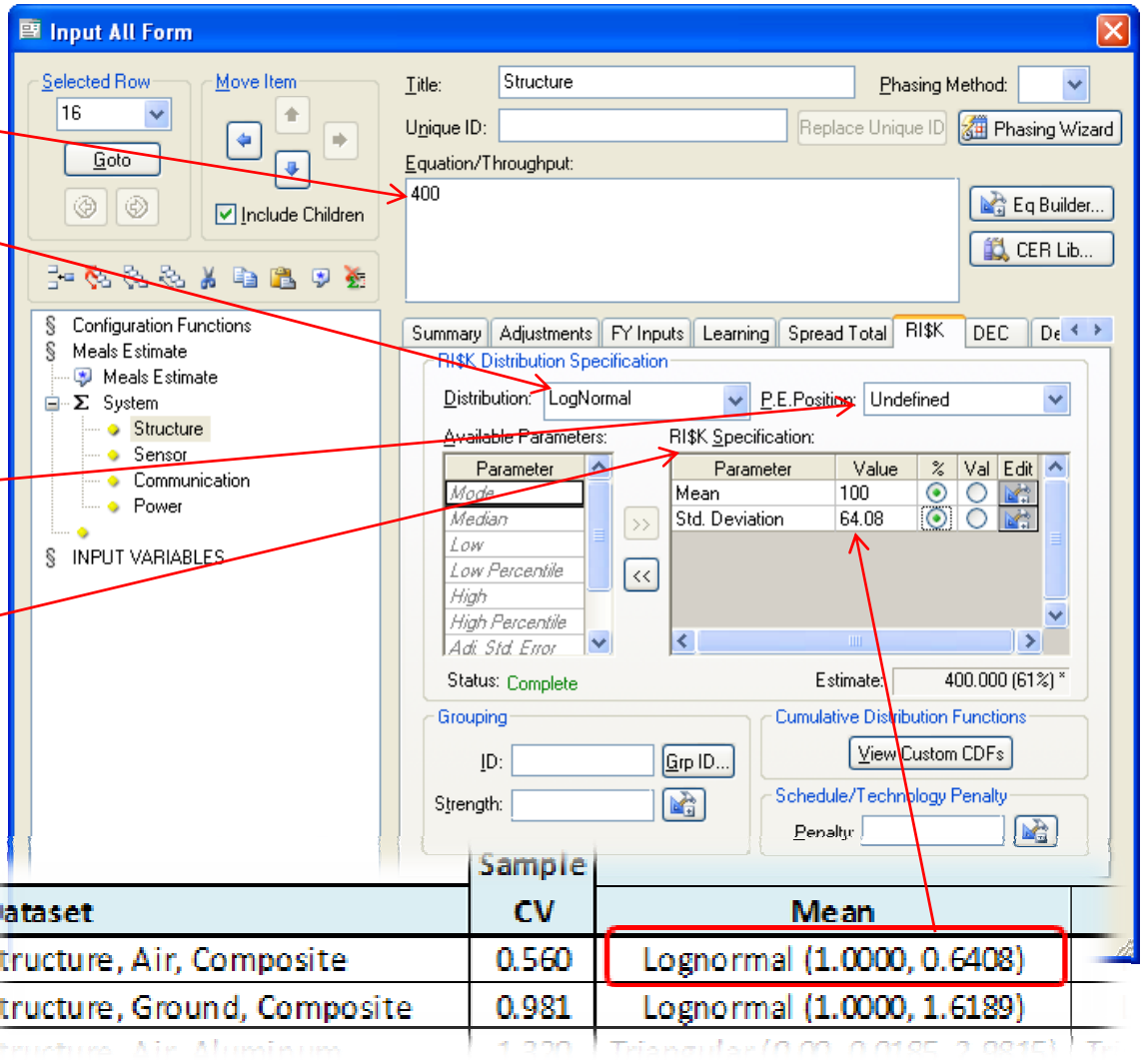
Using a Catalog of Unitized Distributions

- **Locate appropriate table**
- **Select row for your WBS and Class**
- **Select column for the your point estimate interpretation**

	Sample	My Point Estimate is the:		
Dataset	**CV**	**Mean**	**Median**	**Mode**
Sensor, IR	0.490	Normal (1.000, 0.4872)	Normal (1.000, 0.4872)	Normal (1.000, 0.4872)
Sensor, MMW	0.349	Normal (1.000, 0.3490)	Normal (1.000, 0.3490)	Normal (1.000, 0.3490)
Sensor, Laser	0.670	Normal (1.000, 0.6732)	Normal (1.000, 0.6732)	Normal (1.000, 0.6732)
Sensor, Tri-mode	0.855	Beta (0.28, [illegible]2, 0.44, 2.89)	Beta (0.82, 7.42, 0.44, 2.89)	Beta (1.00, 15.07, 0.44, 2.89)
	Sample	My Point Estimate is the:		
Dataset	**CV**	**Mean**	**Median**	**Mode**
Structure, Air, Composite	0.560	Lognormal (1.0000, 0.6408)	Lognormal (1.1969, 0.7602)	Lognormal (6.1976, 1.1691)
Structure, Ground, Composite	0.981	Lognormal (1.0000, 1.6189)	Lognormal (1.9029, 3.0807)	Lognormal (6.8904, 11.1551)
Structure, Air, Aluminum	1.320	Triangular (0.00, 0.0185, 2.9815)	Triangular (0.00, 0.0210, 3.3888)	Triangular (0.00, 1.0000, 16.1310)
Structure, Ground, Aluminum	0.125	Normal (1.0000, 0.1256)	Normal (1.0000, 0.1256)	Normal (1.0000, 0.1256)
	Sample	My Point Estimate is the:		
Dataset	**CV**	**Mean**	**Median**	**Mode**
Communication, UHF XMTR	0.526	Triangular (0.00, 0.6585, 2.3515)	Triangular (0.00, 0.6910, 2.5088)	Triangular (0.00, 1.0000, 3.6310)
Communication, VHF Ground	0.567	Triangular (0.00, 0.0185, 2.9815)	Triangular (0.00, 0.0210, 3.3888)	Triangular (0.00, 1.0000, 16.1310)
Communication, UHF Air	0.350	Normal (1.000, 0.3490)	Normal (1.000, 0.3490)	Normal (1.000, 0.3490)
Communication, UHF Sea	0.423	Triangular (0.00, 1.4385, 1.5715)	Triangular (0.00, 1.3610, 1.4788)	Triangular (0.00, 1.0000, 1.0910)
	Sample	My Point Estimate is the:		
Dataset	**CV**	**Mean**	**Median**	**Mode**
Power, Battery, NiCd	0.594	Beta (0.19, 2.33, 0.87, 1.44)	Beta (0.21, 2.55, 0.87, 1.44)	Beta (0.19, 12.39, 0.87, 1.44)
Power, Battery, Li	0.919	Beta (0.07, 3.87, 0.56, 1.74)	Beta (0.11, 5.47, 0.56, 1.74)	Beta (0.07, 52.02, 0.56, 1.74)
Power, Networks	0.801	Beta (0.07, 2.72, 0.66, 1.20)	Beta (0.08, 3.20, 0.66, 1.20)	Beta (1.00, 38.65, 0.66, 1.20)
Power, Generator	0.600	Lognormal (1.0000, 0.6408)	Lognormal (1.1969, 0.7602)	Lognormal (6.1976, 1.1691)

Input All Form: Lognormal

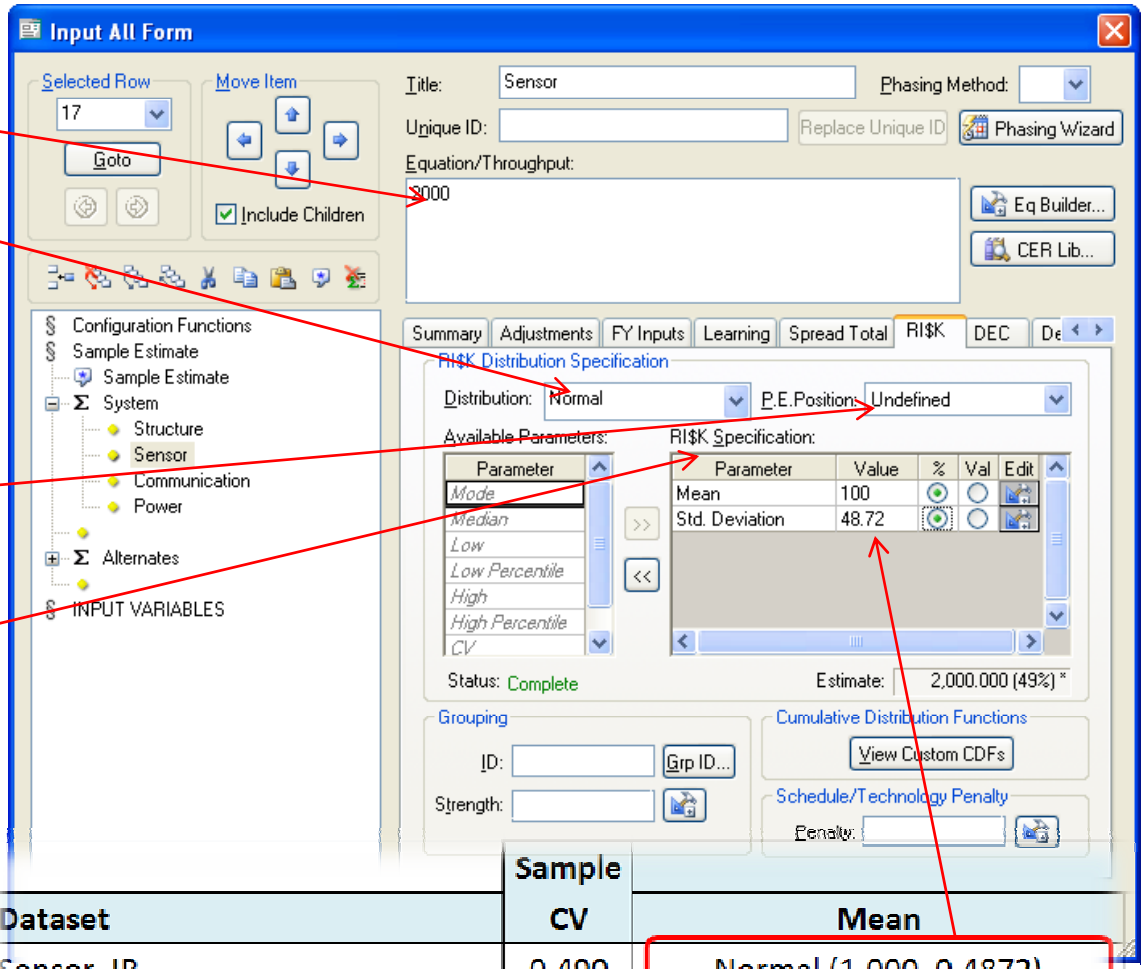
- **Steps in defining a distribution:**
  - **Step 1:** Enter the method or throughput for the row.
  - **Step 2:** Select a distribution type – What is the shape of the uncertainty you want to model?
  - **Step 3:** Enter the Point Estimate (PE) Position – *Use "Undefined" for Unitized Distributions.*
  - **Step 4:** Indentify the remaining shape of the distribution – This will vary depending on the PE position and the bound information you have available to you. **When the distribution is fully specified the status will say Complete.**



Dataset	Sample CV	Mean
Structure, Air, Composite	0.560	Lognormal (1.0000, 0.6408)
Structure, Ground, Composite	0.981	Lognormal (1.0000, 1.6189)

Input All Form: Normal

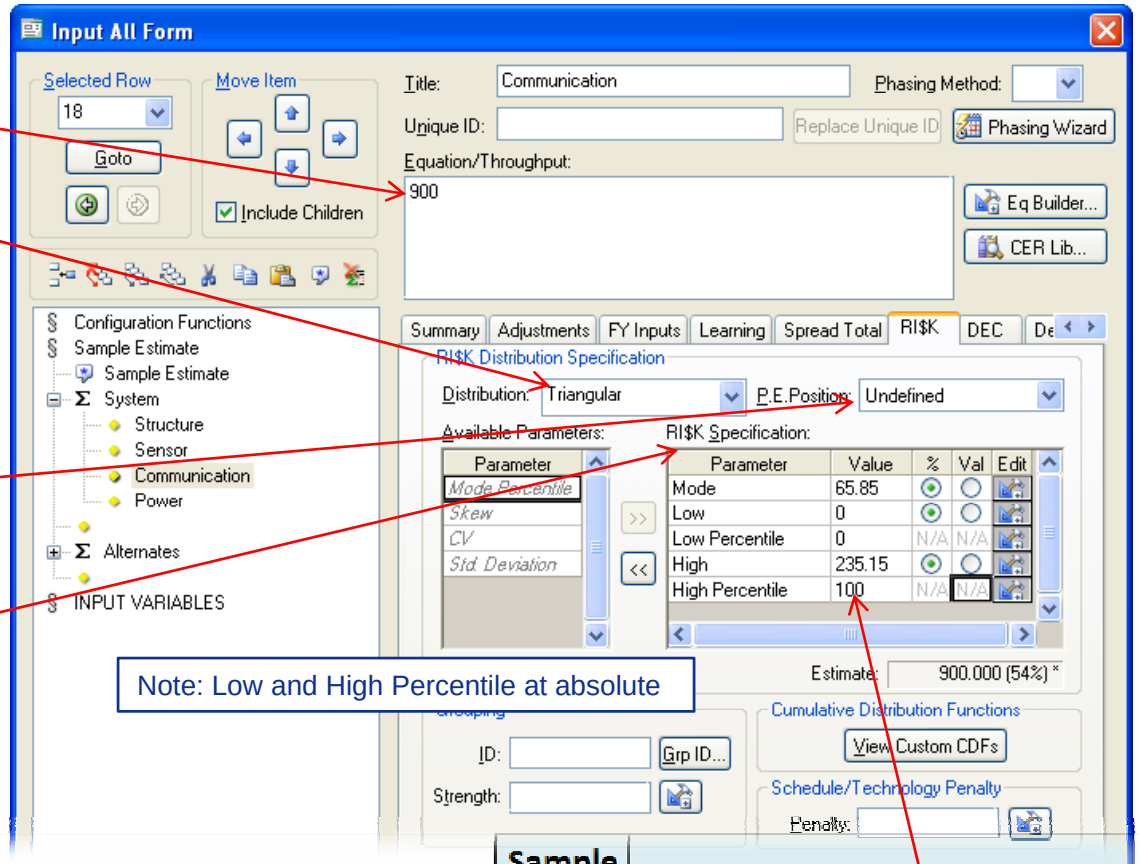
- **Steps in defining a distribution:**
  - **Step 1:** Enter the method or throughput for the row.
  - **Step 2:** Select a distribution type – What is the shape of the uncertainty you want to model?
  - **Step 3:** Enter the Point Estimate (PE) Position – *Use "Undefined" for Unitized Distributions.*
  - **Step 4:** Indentify the remaining shape of the distribution – This will vary depending on the PE position and the bound information you have available to you. **When the distribution is fully specified the status will say Complete.**



Dataset	Sample CV	Mean
Sensor, IR	0.490	Normal (1.000, 0.4872)
Sensor, MMW	0.349	Normal (1.000, 0.3490)

Input All Form: Triangular

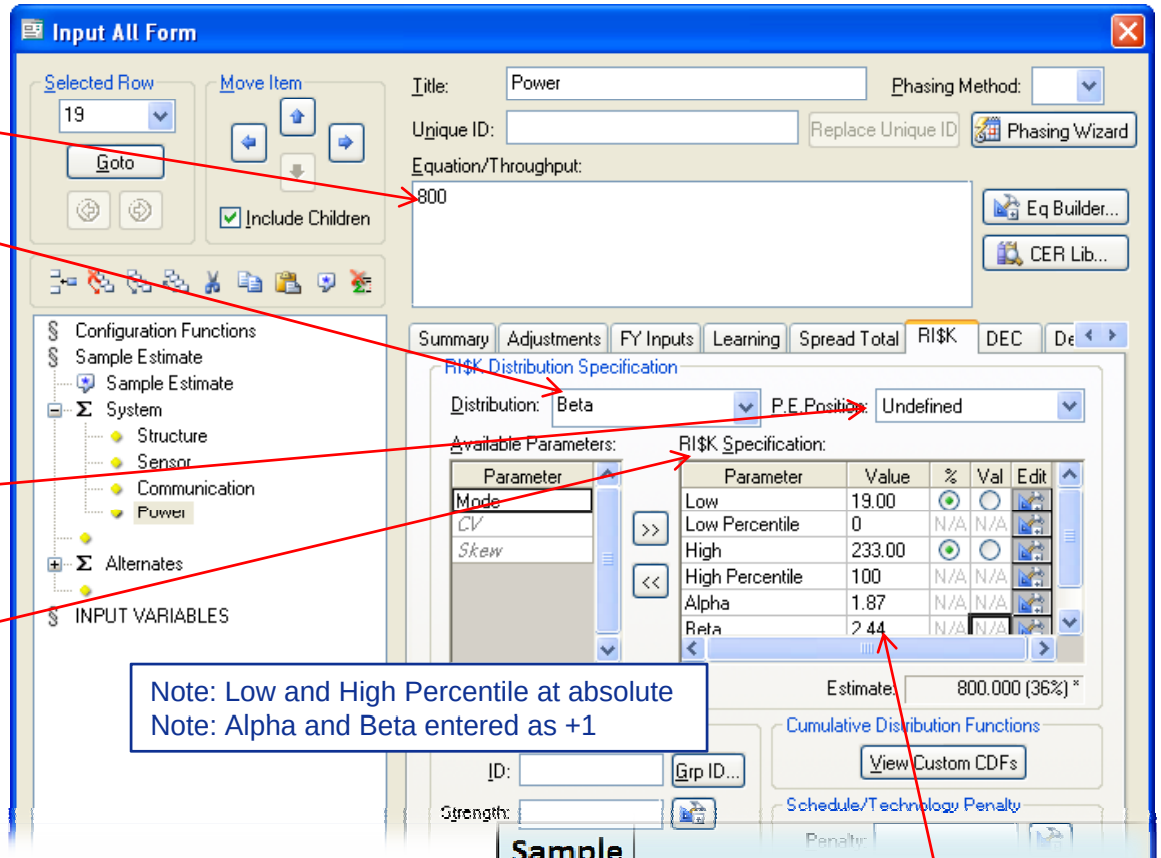
- **Steps in defining a distribution:**
  - **Step 1:** Enter the method or throughput for the row.
  - **Step 2:** Select a distribution type – What is the shape of the uncertainty you want to model?
  - **Step 3:** Enter the Point Estimate (PE) Position – *Use "Undefined" for Unitized Distributions.*
  - **Step 4:** Indentify the remaining shape of the distribution – This will vary depending on the PE position and the bound information you have available to you. **When the distribution is fully specified the status will say Complete.**



Dataset	Sample CV	Mean
Communication, UHF XMTR	0.526	Triangular (0.00, 0.6585, 2.3515)
Communication, VHF Ground	0.567	Triangular (0.00, 0.0185, 2.9815)

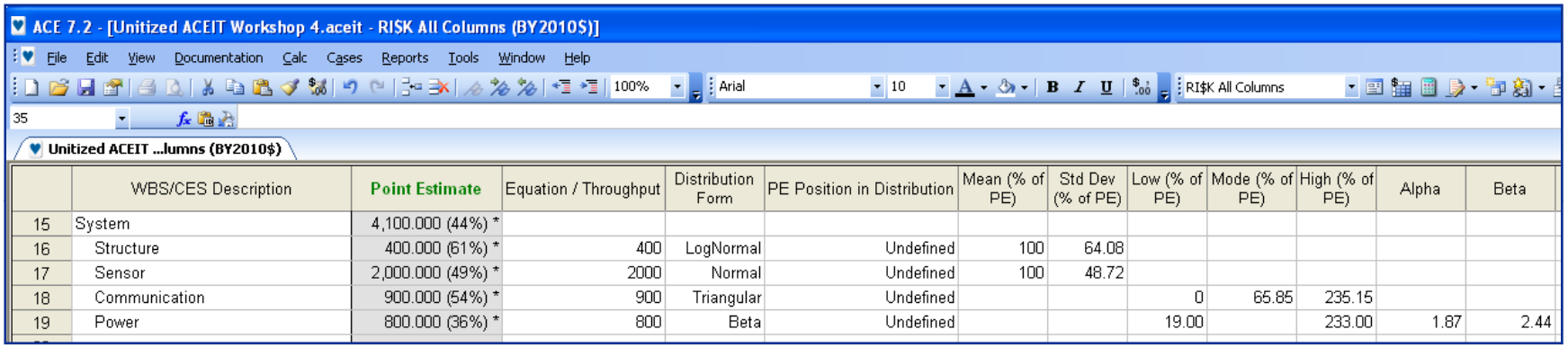
Input All Form: Beta

- **Steps in defining a distribution:**
  - **Step 1:** Enter the method or throughput for the row.
  - **Step 2:** Select a distribution type – What is the shape of the uncertainty you want to model?
  - **Step 3:** Enter the Point Estimate (PE) Position – *Use "Undefined" for Unitized Distributions.*
  - **Step 4:** Indentify the remaining shape of the distribution – This will vary depending on the PE position and the bound information you have available to you. **When the distribution is fully specified the status will say Complete.**



Dataset	Sample CV	Mean
Power, Battery, NiCd	0.594	Beta (0.19, 2.33, 0.87, 1.44)
Power, Battery, Li	0.919	Beta (0.07, 3.87, 0.56, 1.74)

Result



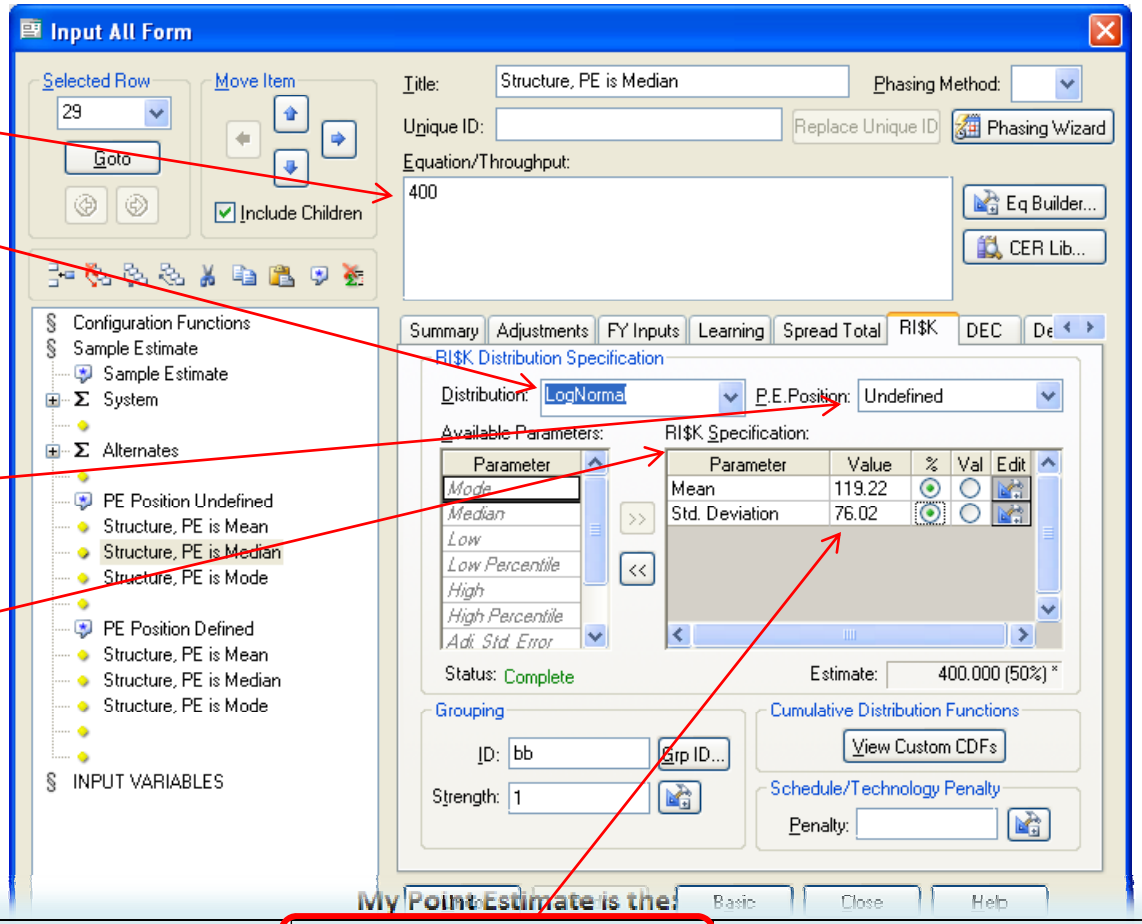
	WBS/CES Description	Point Estimate	Equation / Throughput	Distribution Form	PE Position in Distribution	Mean (% of PE)	Std Dev (% of PE)	Low (% of PE)	Mode (% of PE)	High (% of PE)	Alpha	Beta
15	System	4,100.000 (44%) *										
16	Structure	400.000 (61%) *	400	LogNormal	Undefined	100	64.08					
17	Sensor	2,000.000 (49%) *	2000	Normal	Undefined	100	48.72					
18	Communication	900.000 (54%) *	900	Triangular	Undefined			0	65.85	235.15		
19	Power	800.000 (36%) *	800	Beta	Undefined			19.00		233.00	1.87	2.44

Alternate Point Estimate Positions

- **Each of the preceding four elements assumed the point estimate was the mean.**
- **The next two elements assume the point estimate is the Median and the Mode.**

Input All Form: Lognormal With Median Point Estimate

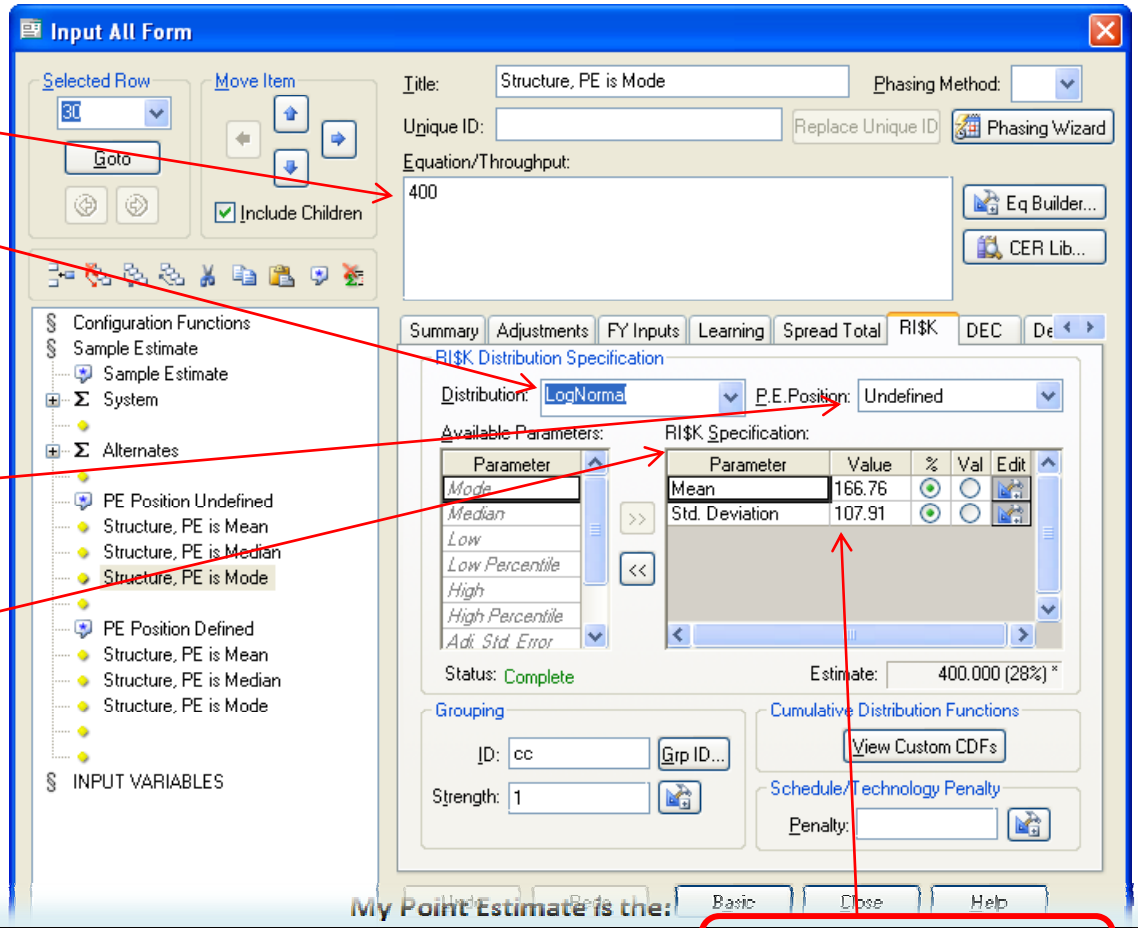
- **Steps in defining a distribution:**
  - **Step 1:** Enter the method or throughput for the row.
  - **Step 2:** Select a distribution type – What is the shape of the uncertainty you want to model?
  - **Step 3:** Enter the Point Estimate (PE) Position – *Use "Undefined" for Unitized Distributions.*
  - **Step 4:** Indentify the remaining shape of the distribution – This will vary depending on the PE position and the bound information you have available to you. **When the distribution is fully specified the status will say Complete**



Dataset	Sample CV	Mean	Median	Mode
Structure, Air, Composite	0.560	Lognormal (1.0000, 0.6408)	Lognormal (1.1922, 0.7602)	Lognormal (1.6676, 1.0791)
Structure, Ground, Composite	0.981	Lognormal (1.0000, 1.6189)	Lognormal (1.9029, 3.0807)	Lognormal (6.8904, 11.1551)

Input All Form: Lognormal With Mode Point Estimate

- **Steps in defining a distribution:**
  - **Step 1:** Enter the method or throughput for the row.
  - **Step 2:** Select a distribution type – What is the shape of the uncertainty you want to model?
  - **Step 3:** Enter the Point Estimate (PE) Position – *Use "Undefined" for Unitized Distributions.*
  - **Step 4:** Indentify the remaining shape of the distribution – This will vary depending on the PE position and the bound information you have available to you. **When the distribution is fully specified the status will say Complete**



My Point Estimate is the:

Dataset	Sample CV	Mean	Median	Mode
Structure, Air, Composite	0.560	Lognormal (1.0000, 0.6408)	Lognormal (1.1922, 0.7602)	Lognormal (1.6676, 1.0791)
Structure, Ground, Composite	0.981	Lognormal (1.0000, 1.6189)	Lognormal (1.9029, 3.0807)	Lognormal (6.8904, 11.1551)

Best practices

Leave the PE Position in Distribution as "Undefined"

- **Retain positive control over model inputs**

ACE 7.2 - [Unitized ACEIT Workshop 5.aceit - RI$K All Columns (BY2010$)]

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Unitized ACEIT ...lumns (BY2010$) | Unitized ACEIT Work...timate, with Risk)

	WBS/CES Description	Point Estimate	Equation / Throughput	Distribution Form	PE Position in Distribution	Mean (% of PE)	Std Dev (% of PE)
27	*PE Position Undefined						
28	Structure, PE is Mean	400.000 (62%) *	400	LogNormal	Undefined	100	64.08
29	Structure, PE is Median	400.000 (50%) *	400	LogNormal	Undefined	119.22	76.02
30	Structure, PE is Mode	400.000 (28%) *	400	LogNormal	Undefined	166.76	107.91
31							
32	*PE Position Defined						
33	Structure, PE is Mean	400.000 (62%) *	400	LogNormal	Mean		64.08
34	Structure, PE is Median	400.000 (50%) *	400	LogNormal	Median		76.02
35	Structure, PE is Mode	400.000 (28%) *	400	LogNormal	Mode		107.91

	Sample	My Point Estimate is the:		
Dataset	CV	Mean	Median	Mode
Structure, Air, Composite	0.560	Lognormal (1.0000, 0.6408)	Lognormal (1.1922, 0.7602)	Lognormal (1.6676, 1.0791)
Structure, Ground, Composite	0.981	Lognormal (1.0000, 1.6189)	Lognormal (1.9029, 3.0807)	Lognormal (6.8904, 11.1551)

A Bad Practice to Avoid

- **Tempting idea: Simplify body of your estimate by defining distributions as Input Variables.**

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Unitized ACEIT ...lumns (BY2010$)

	WBS/CES Description	Unique ID	Point Estimate	Equation / Throughput	Distribution Form	PE Position in Distribution	Mean (% of PE)	Std Dev (% of PE)
14	*Sample Estimate	*Estimate						
15	System		530.000 *					
16	Structures		530.000 *					
17	Fwd Structure		150.000 *	150 * UDIST				
18	Mid Structure		200.000 *	200 * UDIST				
19	Aft Structure		180.000 *	180 * UDIST				
20								
21	*INPUT VARIABLES	*IN_VAR						
22	Structure Unitized Uncertainty Distribution	UDIST	1.000 *	1	LogNormal	Undefined	100	64.08

- **Not recommended as it results in unintended correlation.**

Summary

- Defined and Illustrated Unitized Distributions
- Demonstrated How to Use Unitized Distributions in ACEIT

- Questions